The Globe as a Network: Geography and the Origins of the World Income Distribution

Matthew J. Delventhal[1]

Universitat Autònoma de Barcelona and Barcelona GSE

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Abstract

How important are falling transport costs for patterns of population and income growth since 1000 CE? To answer this question, I build a quantitative dynamic spatial model with an agricultural and a non-agricultural sector, and endogenous fertility, migration, innovation and technology diffusion. In this model there exists an endogenous threshold for global transport costs, which is characterized by a simple network statistic. If transport costs are above this threshold, the world converges to a Malthusian steady state. If transport costs fall below this threshold, the world economy enters a process of sustained growth in population and income per capita. Taking this model to the data, I divide the globe into 2,249 3° by 3° quadrangles. I assign each location an agricultural potential determined by exogenous climate and soil characteristics. I infer bilateral transport costs by calculating the cheapest route between each pair of locations given the natural placement of rivers, oceans and mountains. I calibrate the model so that in the year 1000 the world is in a Malthusian steady state. I then drop the cost of water and land transport exogenously in a way that is consistent with historical evidence and track the endogenous evolution of population and income until the year 2000. Qualitatively, this exercise generates slow but accelerating growth in both population and income per capita for the first 800 years, an abrupt takeoff in growth after 1800 CE with Europe in the lead, and a large increase in the dispersion of income per capita after 1800 CE. Quantitatively, the model accounts for 55% of the variation in population density across 10 major regions in 1000 CE, 44% of the variation in income per capita across regions in 1800 CE, and is able to generate 43% of the overall dispersion in income per capita in 2000 CE.

Keywords: Geography, trade, diffusion, structural change, networks.

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1 Introduction

Over the past 1000 years, successive improvements in transportation technology have given people in every part of the world progressively easier access to goods, ideas, and people from every other part of the world. During this same period, the world has experienced gradually accelerating growth in population, and an abrupt increase in income per capita growth, first in Europe and then in other regions, after 1800 CE. This latest burst of growth is the proximate cause of the distribution of income across regions we see in the world today, with the great distance between rich and poor countries and all the challenges and opportunities this entails.

How big is the role of falling transport costs in these great shifts of population and income? Why did the growth rate of income per capita increase abruptly around 1800 CE, and why in some places and not in others? These are the questions I address in this paper. To that end, I build a quantitative dynamic spatial model, with an agricultural and a non-agricultural sector. In this model, I allow both population, through fertility and migration, and knowledge, through innovation and diffusion, to be fully endogenous. Bilateral transport costs between each pair of locations determine the cost of trade, the cost of migration, and the speed of the diffusion of ideas. These shape the networks of trade, migration and technology diffusion through which outcomes in distinct locations are linked. Productivity in the agricultural sector depends on exogenous factors such as climate and soil characteristics, while the productivity of the non-agricultural sector depends on access to stocks of ideas.

I find that this model implies the existence of a threshold for global transport costs, which can be characterized in terms of a simple network statistic. If transport costs are above this threshold, population growth drives down income per capita, and the world converges to a Malthusian steady state with no growth. If transport costs fall below this threshold, population growth leads to a structural transformation from the agricultural to the non-agricultural sector, and the world economy enters a process of sustained growth in population and income per capita. In general, a universal reduction in transport costs will impact some locations more than others, so the take-off into growth may occur in a subset of locations at first. Trade and technology diffusion imply that all locations will start to catch up eventually.

Taking this model to the data, I divide the world into 3° by 3° quadrangles. I exclude quadrangles that contain no land or that are in Antarctica, leaving 2,249 habitable locations. I assign each location an agricultural potential based on available evidence from ecological studies. I infer bilateral transport costs by calculating the cheapest route between each pair of locations, given the natural placement of rivers, oceans and mountains, and given the cost of traversing each of these topographical features.

I then conduct a quantitative exercise in two stages. First, I calibrate the handful of parameters that are not already taken from historical data or tied to specific targets so that model predictions for population density in all of the 2,249 locations match the data for 1000 CE as closely as possible, under the assumption that the world is in a Malthusian steady state. I then reduce the costs of water and land transport gradually, in a way that is consistent with historical evidence, and track the endogenous evolution of population and income in 50 year periods until 2000 CE.

Qualitatively, this exercise is able to match all of the salient features of the data. The model generates slow but accelerating growth for the first 800 years, an abrupt takeoff around 1800 CE with Europe in the lead, and a large increase in the dispersion of income per capita across regions after 1800 CE.

Quantitatively, the model is able to account for most of the variation in population density across 10 major regions in 1000 CE–55% in all. China, India and Europe were more densely populated than other regions because they had more land with better agricultural potential better-linked by water transport. Europe is particularly well-connected to water transport, and so it benefits from the water-biased transport cost reductions that occur before 1750 CE. This is why Europe starts growing first, and is what allows the model to account for nearly half (44%) of the variation in income per capita across regions in 1800 CE, the first year for which there exists meaningful data. The model tracks the sharp rise of dispersion in the distribution of income per capita during the 19th century almost perfectly, and ultimately generates 43% of the overall dispersion across regions in the 2000 CE.

There are also some patterns that the model is not able to match. In particular, the model does not predict enough growth in the United States, Canada, Australia and New Zealand after 1800 CE. Also, the model predicts too much convergence between Europe and the rest of the world during the 20th century. I believe that these observations indicate avenues for future research, and I discuss them in more detail in the conclusion of the paper.

This study breaks new ground in a number of areas. To the best of my knowledge, it is the first study to propose a theory of the take-off from stagnation to growth as a global phenomenon dependent on a reduction in transport costs. It is related to the theory of Desmet and Parente (2012), who examine the role of market size in the industrial revolution. I build upon this study by considering the role of transport costs, and expanding the analysis to a global scope. It is also related to Galor and Weil's (2000) unified growth theory. I build upon their study by considering the role of space, and by providing a particular rationale for the relationship between technological progress and population size that they propose. In my model, when transport costs are reduced, we might also say that the effective population size has increased, as people living in different locations have been brought effectively closer together. So when transport costs fall below the critical level, we could also say that a "critical mass" of connected people has been created, not unlike the threshold population size which emerges from Galor and Weil's model.[2]

This is also the first study to leverage available data on topography and exogenous climate and soil characteristics in a quantitative model to assess their role in determining the distribution of population and income in the world today. Prominent among previous efforts to assess the impact of geographical features on the distribution of population and income are Henderson, Squires, Storeygard and Weil (2016) and Gallup, Sachs and Mellinger (1999). I confirm the main conclusions of these studies in finding an association between agricultural potential and high pre-modern population density, and between access to water transport and modern growth, and propose and test quantitatively specific mechanisms through which these features can have an impact. Also, these studies implicitly assume that the value of access to a river or to the coast is the same in every location in the world, regardless of how far away or how wealthy potential trading partners are. The method that I use, which, similar to Donaldson and Hornbeck (2016), calculates distances to trading partners and determines the value of the trading connection using a general equilibrium model, accounts better for this natural heterogeneity.

This study is also, to my knowledge, the first to allow for endogenous population growth in a spatial setting. A recent study which analyzes the global distribution of population and income using a spatial dynamic framework is Desmet, Nagy and Rossi-Hansberg's (2016). In contrast to my focus on understanding how we arrived at the distributions of 2000 CE, they take these distributions as a starting point, and run counterfactual scenarios for the future. Population growth plays no role in their model. Another related paper in this vein is that of Nagy (2017), which takes aggregate population and technology growth in the 19th century United States as given, and seeks to explain their distribution across space in the decades leading up to 1860.[3]

This study is also related to the literature which has looked at the relationship between between market access and the global distribution of income. Redding and Venables (2004) and Head and Mayer (2011) find important static effects, taking the current distribution of population and technology as given. The current study extends these efforts by investigating the role of market access in determining these distributions. There have also been a number of studies measuring the importance of market access within a single country, such as Donaldson and Hornbeck (2016).

My paper is also related to efforts such as Alcalá and Ciccone's (2004) and Pascali's (2016) to assess the impact of trade on growth. Pascali's study is particularly related, as he exploits heterogeneity in access to water transport in a similar fashion, although in

[2]Galor and Mountford (2008) also analyze the effect of increased trade on the transition from stagnation to growth, and in particular on the divergence in income per capita between the richest and poorest countries. They argue that globalization accelerated the transition to sustained growth in more advanced countries, and delayed it in less-advanced countries.

[3]Similarly, the dynamic spatial framework of Caliendo, Dvorkin and Parro (2017) takes population growth as given.

his case he uses it to construct an instrumental variable. Whereas studies in this strain of literature have been primarily interested in establishing whether or not there is an effect of trade on growth, I build on their insights by proposing a particular model of this relationship and assessing its performance quantitatively.

Similarly to this paper, Buera and Oberfield (2015) propose diffusion as a dynamic gain from trade. I build upon their insights by modeling this mechanism in a spatial setting and assessing its impact on growth over the last 1000 years quantitatively. Comin, Dmitriev and Rossi-Hansberg (2013) propose a similar model of the diffusion of technology across space, and show that it is consistent with observed patterns of technology diffusion over the past 150 years. My setup differs that of Nagy (2017) and Desmet, Nagy and Rossi-Hansberg (2016) in that I track to transmission of ideas to particular locations, which can then themselves transmit the idea, as if it were a virus.

Finally, my study builds on that of Acemoglu, Johnson and Robinson (2005), who document that western Europe's higher rate of growth between 1500 and 1800 is almost entirely due to the growth of a handful of countries on the Atlantic Ocean who were engaged in substantial overseas trade. While Acemoglu and coauthors emphasize the role of institutions in deciding which of the Atlantic traders were best able to take advantage of their ocean access, my paper confirms and deepens the significance of the first fact, by showing that falling water transport costs during this period benefited some locations more than others and can account quantitatively for a number of key patterns in population and income growth.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the long run outcomes of the model. Section 4 describes how I bring the model to the data in a quantitative exercise. Section 5 presents and discusses the results of the calibration of the initial 1000 CE steady state. Section 6 presents and discusses the results of the simulation of the evolution of global population and income per capita from 1000-2000 CE. Section 7 discusses possible extensions and concludes.

2 Theoretical framework

The basic building blocks are as follows. Time is discrete, and indexed by t. Each model period is intended to represent a span of about 50 years. There exist a finite number of discrete locations n, contained in the set $N \equiv \{1, 2, ..., n\}$. Each location, at each point in time, is distinguished by three permanent, exogenous characteristics, and two endogenous characteristics that evolve over time. The three exogenous, permanent characteristics are $\lambda_i > 0$ for $i \in N$, the quantity of available land, $\alpha_i \geq 0$ for $i \in N$, agricultural potential, and bilateral transport costs reflected in $\gamma_{ij} \in [0, 1]$ for $i, j \in N$. The two endogenous, time-varying characteristics for are $x_i(t) \geq 0$ for $i \in N$, the number of residents, and $m_i(t) \geq 0$ for $i \in N$, the stock of ideas.

Consumers are endowed with labor, from which they derive wage income, and value goods and housing. There are many types of goods, and firms produce each one using labor, land and other goods as inputs. There is one type of housing, and producing it requires land and goods. Housing production is more land-intensive than goods production, and the demand for it increases the negative welfare effects of having many consumers living in a single location.[4]

All the varieties of goods exist in a continuum, and are indexed between 0 and 1. Among these, there are two basic categories or sectors. All of the goods indexed between 0 and $A < 1$ (the span $[0, A]$) are agricultural goods. All of the goods indexed between A and 1 (the span $(A, 1]$) are non-agricultural goods.

All goods may be produced in all locations, but different locations are better at producing some goods than others. This means that consumers and firms in different locations can gain by trading with each other, each specializing in the type of production they excel at.

Locations may also differ in their average suitability for producing agricultural and non-agricultural goods. Average agricultural suitability is partly determined by the exogenous, time-invariant characteristics of each location which are summarized by agricultural potential α_i. This quantity is meant to represent all of the durable climatic and geological characteristics that make some places respond more fruitfully to the efforts of the farmer.

Average non-agricultural suitability does not directly depend on any fixed, exogenous feature of a location. Instead it depends on the endogenous, time-varying stock of ideas, $m_i(t)$. The way in which this stock evolves over time is as follows. Firms that produce goods employ labor and land in innovation, which gives them an immediate, private productivity boost. As an externality, this innovative effort also leads to the discovery of new ideas. These ideas are added to local stock of ideas, and may also diffuse between locations that are trading partners.

If the stock of ideas in a location is small, its overall productivity will be mostly determined by its agricultural potential. But if the stock of ideas grows, the importance of this exogenous characteristic will decline.

Trade is limited by the cost of transporting goods. Bilateral transport costs are embodied in the parameters $\gamma_{ij} \in [0, 1]$, which each represent the fraction of goods sent from i to j that arrive. It is assumed that transport within a location is costless ($\gamma_{ii} = 1$) and that the triangle inequality holds ($\gamma_{ij}\gamma_{jk} \leq \gamma_{ik}$ for $\forall i, j, k$). In the current section we postpone the analysis of time-varying transport costs, and assume that transport costs are constant over time.

Transport costs determine the trade opportunities available to consumers and firms in each location. They also determine, according to simple functions, the strength of two

[4] Also known as congestion effects.

other types of bilateral links. These are the cost faced by consumers when migrating between locations, and the probability that an idea invented in one location will spread to another. Therefore, locations which have more trading opportunities will also learn about more new ideas, sooner, and be more easily reached by migrants.

Consumers are atomistic and live a single period. The number of consumers living in each location, $x_i(t)$, is determined according to two processes. The first process is fertility. For each consumer who lived in a location the previous period, a certain number will be born there the current period. Á la Hansen and Prescott (2002), the fertility rate is determined as a simple function of parents' real income, which is a measure of the abundance of goods and housing they enjoyed. If goods and housing are very scarce, net fertility will be negative and the local population will shrink. If they are abundant enough, it will be positive and the local population will grow.

The second process determining the distribution of consumers across locations is migration. Given their birthplace, each consumer chooses either that location or another in which to work and consume. They will tend to move towards locations where there is a greater abundance of goods and housing, but to do so they must pay a migration cost. They also have idiosyncratic preferences for specific locations, which may cause a minority of individuals to choose locations which are less desirable in terms of real income and migration cost.

In the following subsections, I will specify each component of the framework in greater detail. I will also derive the equilibrium conditions and laws of motion that jointly determine current real income in each location, population growth, and the invention and diffusion of technology over time.

Many of the choices and processes that will be described take place in the context of a single time period. Therefore, for simplicity, I will from here on omit t-indices except where doing so introduces ambiguity.

2.1 Consumers

Consumers are atomistic and live a single period. The number of consumers born in each location at time t is denoted by $x_{i,b}(t)$, hereafter referred to as $x_{i,b}$, except where ambiguous. The first decision they must make is where to live, work and consume. The number of consumers choosing each location at time t is denoted by $x_i(t)$, hereafter referred to as x_i, except where ambiguous.

A consumer born in $j \in N$ chooses a location $i \in N$ in which to live based on three factors. First, their beliefs about the real income they can enjoy in each location, u_i^*. Second, moving costs between their birthplace and each destination. It is assumed that in order to move, consumers must give up a certain fraction of the real income they will earn at their destination. The inverse moving cost, $\vartheta_{ji} \in [0,1]$, represents the fraction of real income that they get to keep. Third, consumers have random idiosyncratic preferences for each potential destination, represented by $\mu_i \sim F_i(\mu)$, drawn independently across individuals and locations from cumulative distribution function $F_i(.)$.

Formally, the location choice problem of a consumer born in $j \in N$ is given by

$$\max_{i \in N} \{\mu_i \vartheta_{ji} u_i^*\} \, |j \in N \tag{1}$$

In equilibrium, consumers' ex-ante beliefs must be true, and coincide with ex-post real income, $u_i^* = u_i$. Inverse moving costs are a simple function of transport costs,

$$\vartheta_{ij} = \zeta_{m,0} \gamma_{ij}^{\zeta_{m,1}},$$

for some $\zeta_{m,0} \in [0,1]$ and $\zeta_{m,1} \geq 0$. Idiosyncratic preference shocks are drawn from a Fréchet distribution, so that

$$F_i(\mu) = e^{-\mu^{-\varkappa}},$$

for $\varkappa > 1$.

Upon arriving in their destination $i \in N$, the choices consumers make of how much to consume of housing and of each good may be characterized in terms of a representative consumer. The representative consumer's real income is determined by their consumption of goods and housing as given by

$$u_i = \left(\int_0^1 c_{i,l} dl \right)^{\frac{\alpha}{\rho}} h_i^{1-\alpha}, \tag{2}$$

where $c_{i,l}$ represents the quantity consumed of good $l \in [0,1]$, and h_i represents the quantity of housing consumed. Parameter $\alpha \in [0,1]$ determines the importance of housing relative to goods consumption, and $\rho \in [0,1]$ determines the elasticity of substitution between different goods.

Each consumer is endowed with 1 unit of labor, which they provide inelastically to the local market in exchange for prevailing wage w_i. It assumed that the rights to land are distributed equally among all residents of location i, so that each owns a quantity $\frac{\lambda_i}{x_i}$. Given land rents $p_{i,\lambda}$, each consumer's income is equal to $w_i + p_{i,\lambda} \frac{\lambda_i}{x_i}$. The representative consumer's budget constraint is then given by

$$\int_0^1 p_{i,l} c_{i,l} dl + p_{i,h} h_i = w_i + p_{i,\lambda} \frac{\lambda_i}{x_i}, \tag{3}$$

where $p_{i,l}$ represents the local equilibrium price of good $l \in [0,1]$ and $p_{i,h}$ represents the equilibrium price of housing.

Given i, then, the problem of the consumer is maximize (2) subject to (3).

2.2 Goods Firms

Firms may enter freely into the production of any good $k \in [0,1]$ in any location $i \in N$ with zero fixed cost. Let us assume for the moment, as we will later confirm, that the problem of the producers of each good in each location may be characterized in terms of a representative producer.

The suitability of each location i, for producing each good $k \in [0,1]$, at each point in time t, is determined by productivity shock $s_{i,k}(t)$, hereafter referred to as $s_{i,k}$, which is drawn independently across locations, goods, and time periods. Each location has two distributions of productivity shocks, one for agricultural goods, and one for non-agricultural goods. For agricultural goods, $k \in [0, A]$, $s_{i,k} \sim G_{i,a}(s)$ with Fréchet cumulative distribution function $G_{i,a}(.)$ being given by

$$G_{i,a}(s) = e^{-\lambda_i \alpha_i^{\chi} s^{-\chi}}, \tag{4}$$

for $\chi > 1$. For non-agricultural goods, $k \in (A, 1]$, $s_{i,k} \sim G_{i,n}(s,t)$, with

$$G_{i,n}(s,t) = e^{-\lambda_i m_i(t)^{\chi} s^{-\chi}}, \tag{5}$$

where $m_i(t)$ represents the time-t stock of ideas in location i and will hereafter be referred to as m_i. It is necessary to factor land area into the expectations of these distributions, because fundamentally they are meant to represent spatial variation in suitability for producing different goods, and locations in this model are allotted different amounts of space. The properties of the Fréchet distribution mean that $G_{i,a}(.)$ and $G_{i,n}(.)$ can be derived by assuming that a shock for each good is drawn for each tiny piece of land from $G^*_{i,a}(s) = e^{-\alpha_i^{\chi} s^{-\chi}}$ and $G^*_{i,n}(s) = e^{-m_i^{\chi} s^{-\chi}}$, and that the best draws are used. $G_{i,a}(.)$ and $G_{i,n}(.)$ then reflect the maximum of draws from $G^*_{i,a}(.)$ and $G^*_{i,n}(.)$ across the λ_i units of land available in i.

After observing $s_{i,k}$, the first choice made by the representative producer of k in i is how much to innovate. By employing labor $b_{i,k,I}$ and land $l_{i,k,I}$, the firm is able to improve its own efficiency in the current period. Final efficiency is given by

$$\hat{s}_{i,k} = s_{i,k} \left(b_{i,k,I}^{\eta} l_{i,k,I}^{1-\eta} \right)^{\kappa}, \tag{6}$$

where $\eta, \kappa \in [0,1]$.

Then, taking $\hat{s}_{i,k}$ as given, each firm chooses the quantity of labor, land and intermediate inputs to employ in production. The quantity produced $q_{i,k}$ is determined according to

$$q_{i,k} = \hat{s}_{i,k} \left(b_{i,k}^{\eta} l_{i,k}^{1-\eta} \right)^{1-\sigma-\kappa} \left(\int_0^1 z_{i,k,l}^{\rho} dl \right)^{\frac{\sigma}{\rho}}, \tag{7}$$

where $b_{i,k}$, $l_{i,k}$ and $z_{i,k,l}$ for $l \in [0,1]$ represent the quantities of labor, land and intermediate inputs employed, and $\sigma \in [0, 1-\kappa]$ is a parameter. Note that the production function, including the investment in innovation, exhibits constant returns to scale overall. This allows the representative firm characterization, and, together with the assumption of free entry and zero fixed cost, implies that firms must earn zero profits in equilibrium.

As firms must earn zero profits in the end, the firm's profit maximization problem can be fully represented as one of cost minimization, taking prices and the market-clearing quantity $q_{i,k}$ as given. Formally, the problem of the firm is

$$\min_{b_{i,k,I}, b_{i,k}, l_{i,k,I}, l_{i,k}, z_{i,k,l}} \left\{ w_i \left(b_{i,k,I} + b_{i,k} \right) + p_{i,\lambda} \left(, l_{i,k,I} + l_{i,k} \right) + \int_0^1 p_{i,l} z_{i,k,l} dl \right\}, \tag{8}$$

subject to (7) and (6).

The zero-profit condition implies that in equilibrium all firms must have a cost of production inversely related to their efficiency shock and equal to $\frac{P_i}{s_{i,k}}$, where P_i is defined as the efficiency price of a unit of output in location i. When selling its output to a buyer in some location $j \in N$, zero profits implies that the price charged will be $\frac{P_i}{s_{i,k}\gamma_{ij}}$, just covering the costs of production and transport.

2.3 Housing Firms

Firms may also enter freely into the production of housing in any location with zero fixed cost. The representative housing firm employs a quantity of land $l_{i,h}$ and quantities of intermediate inputs $z_{i,h,l}$ for $l \in [0,1]$ to produce a quantity of housing H_i according to

$$H_i = \left(\int_0^1 z_{i,h,l}^{\rho} dl \right)^{\frac{\varphi}{\rho}} l_{i,h}^{1-\varphi}. \tag{9}$$

In equilibrium, the profits earned by this housing producer must be zero. Taking the market-clearing quantity of housing H_i as given, the problem of the housing firm is

$$\min_{l_{i,h}, z_{i,h,l}} \left\{ \int_0^1 p_{i,l} z_{i,h,l} dl + p_{i,\lambda} l_{i,h} \right\} \tag{10}$$

subject to (9).

2.4 Market Equilibrium

When considering equilibrium outcomes in this economy, the first thing we need to know is the vector of real incomes, u_i for $i \in N$, which will result in a single period from any given allocation of population and idea stocks. To that end, let us define a market equilibrium as follows.

Given resident populations x_i and idea stocks m_i, a market equilibrium is defined as prices for goods, land, and labor, production decisions by goods firms and housing firms, and consumption decisions by consumers, such that markets for goods, land and labor clear, and all decisions are optimal.

As is shown in appendix A.3, these equilibrium conditions imply that real income in location i depends on two key quantities. The first is population density, $\frac{x_i}{\lambda_i}$. The second is a measure of location i's trade access to highly productive locations, which we will call market access. Market access is defined as $\mathbb{M}_i \equiv \left[\int_0^1 \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}}$. In equilibrium it is equal to the following weighted sum:

$$\mathbb{M}_i = B_M \left[A \sum_{j\in N} \left(\frac{P_i}{P_j}\right)^{\chi} \gamma_{ji}{}^{\chi}\alpha_j + (1-A)\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}{}^{\chi} m_j\right]. \tag{11}$$

In the above equation, B_M represents a constant equal to $\Gamma\left(1-\frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}}$, where $\Gamma(.)$ denotes the gamma function. What (11) means is that market access is improved by having low transport-cost access to locations that have high agricultural potential, large stocks of ideas, and low costs of production.

Equilibrium real income as a function of population density and market access is given by the following:

$$u_i = B_u \left(\frac{\lambda_i}{x_i}\right)^{\nu_2} \mathbb{M}_i^{\nu_1}, \tag{12}$$

where

$$\nu_1 \equiv \frac{\alpha + (1-\alpha)\varphi}{\chi(1-\sigma)},$$
$$\nu_2 \equiv 1 - \eta[\alpha + (1-\alpha)\varphi],$$

and

$$B_u \equiv \alpha^{\alpha}\left(\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\right)^{\alpha+\varphi(1-\alpha)}\left(\frac{\varphi(1-\eta)}{1-\varphi}\right)^{\varphi(1-\alpha)}\frac{(1-B_{g,\lambda})^{1-\alpha}\left[\eta B_{g,\lambda}+1-\eta\right]^{\alpha}}{B_{g,\lambda}^{\eta[\alpha+\varphi(1-\alpha)]}}.$$

and $B_{g,\lambda} = \frac{(1-\eta)(\alpha+\varphi(1-\alpha))}{(1-\varphi)(1-\alpha)+(1-\eta)(\alpha+\varphi(1-\alpha))}$ represents the constant equilibrium fraction of land dedicated to goods production.

2.5 Evolution of population

Population evolves over time through two processes: fertility and migration. Following Hansen and Prescott (2002), net fertility is assumed to be a simple function $f(u)$ of parents' real income. It is assumed that $f(u)$ satisfies two properties: First, that if real income is low enough, population growth is negative. Second, that as real income increases without bound, that fertility approaches a finite positive limit. Formally, these two conditions can be represented as $\lim_{u \to 0} = 0$, and $\lim_{u \to \infty} = \bar{u}$, for some $\bar{u} \geq 1$.

The quantity of consumers born in a location, as a function of the number of consumers who lived there the previous period, is given by

$$x_{i,b}(t) = x_i(t-1) f(u_i(t)) \tag{13}$$

Migration occurs as the result of consumer choices of where to live, given their place of birth. The properties of the Fréchet distribution allow the following characterization of the fraction of consumers who will choose to move from i to j:

$$l_{ji} = \frac{\vartheta_{ji}^{\varkappa} x_{i,b} u_i^{\varkappa}}{\sum_{k \in N} \vartheta_{jk}^{\varkappa} x_{k,b} u_k^{\varkappa}} \tag{14}$$

The number of consumers living in i as a function of the numbers of consumers born in every location is then equal to

$$x_i = \sum_{j \in N} l_{ji} x_{j,b} \tag{15}$$

Combining (13) and (15) yields the following law of motion for $x_i(t)$:

$$x_i(t) = \sum_{j \in N} l_{ji}(t) x_j(t-1) f(u_j(t)) \tag{16}$$

2.6 Evolution of technology

Technological progress happens as a result of the resources that firms spend in innovation. Each firm benefits privately from its innovative effort by through an immediate increase in productivity. As an externality, the labor $b_{i,k,I}$ and land $l_{i,k,I}$ that each firm dedicates to innovation leads to the discovery of a number of ideas equal in measure to $b_{i,k,I}^{\eta\phi} l_{i,k,I}^{1-\eta\phi}$. The parameter $\phi > 0$ determines whether returns to density of innovative activity are increasing, decreasing, or constant. Aggregating across firms, the total number of ideas discovered in location i at time t is given by

$$\hat{m}_i(t) = B_m x_i(t)^{\eta\phi} \lambda_i^{1-\eta\phi}, \tag{17}$$

where $B_m \equiv B_{g,\lambda}^{1-\eta\phi} \frac{\kappa}{1-\sigma}$. These ideas are added to the location-i stock of ideas at time $t+1$.

Each period, any idea already in the stock of ideas at the start of period t in location

i has a probability $\bar{\theta}_{ij}$ of diffusing to each other location j. If the idea diffuses and is not already known in location j, then it is added to location j's stock of ideas at time $t+1$. The diffusion probabilities are determined as a simple function of transport costs, according to

$$\bar{\theta}_{ij} = \gamma_{ij}^{\zeta_d} \tag{18}$$

for $\zeta_d > 0$.

A particular idea, discovered in a particular location i, may arrive in another location j after only one period, or after two, three, or more periods. It may be transmitted directly, or it may be transmitted through an intermediate chain of other locations that learn the idea first. To model this process, let $\theta_{ij,s}$ for $s \in \{0, 1, 2, ...\}$ represent the probability that an idea invented in i is known in j after s periods. In the period of its discovery an idea is known in its home location and not in any other, so $\theta_{ii,0} = 1$ and $\theta_{ij,0} = 0$ for $i \neq j$. For $s \geq 1$, $\theta_{ij,s}$ is determined by the following recursive process:

$$\theta_{ij,s} = 1 - \underbrace{\underbrace{(1 - \theta_{ij,s-1})}_{\text{Pr not known at } s-1} \underbrace{\prod_{k \in N} \left(1 - \theta_{ik,s-1}\bar{\theta}_{kj}\right)}_{\text{Pr no arrival in period } s}}_{\text{Pr not known at } s}. \tag{19}$$

In the above expression, $1 - \theta_{ij,s-1}$ is the probability that the idea has not already reached j before s periods have passed; $\theta_{ik,s-1}\bar{\theta}_{kj}$ is the probability of transmission from k to j during the current period; and $\prod_{k \in N} \left(1 - \theta_{ik,s-1}\bar{\theta}_{kj}\right)$ is the probability that no location transmits the idea to j during the current period. Clearly, as long as there exists some sequence of $m+2$ locations, $\{i, l_1, l_2, \ldots, l_{m-1}, l_m, j\}$, such that the product $\bar{\theta}_{il_1}\bar{\theta}_{l_1 l_2} \ldots \bar{\theta}_{l_{m-1} l_m}\bar{\theta}_{l_m j} > 0$, then $\lim_{s \to \infty} \theta_{ij,s} = 1$. In other words, as long as there is a path of finite distance, however long and indirect, from i to j, then all ideas discovered in i will eventually arrive in j.

Each period after its discovery, each idea faces a probability $\delta \in [0, 1]$ of becoming obsolete and no longer contributing to the level of technology in any of the locations in which it is known. Thus, the time-t level of technology in location i, $m_i(t)$, can be expressed as the following function of the ideas that have been discovered in each location in each previous period:

$$\begin{aligned} m_i(t) &= \hat{m}_i(t) + \sum_{s=1}^{\infty}(1-\delta)^s \hat{m}_i(t-s) + \sum_{s=1}^{\infty}(1-\delta)^s \sum_{j \neq i} \theta_{ji,s}\hat{m}_j(t-s) \\ &= \sum_{s=0}^{\infty}(1-\delta)^s \sum_{j \in N} \theta_{ji,s}\hat{m}_j(t-s) \end{aligned} \tag{20}$$

Rearranging (20), it is also possible to write the following law of motion:

$$m_i(t) = (1-\delta)m_i(t-1) + \hat{m}_i(t) + \sum_{s=1}^{\infty}(1-\delta)^s \sum_{j\neq i} \left[\theta_{ji,s} - \theta_{ji,s-1}\right] \hat{m}_j(t-s) \qquad (21)$$

3 The Long Run

In the previous section, I constructed a model in which flows of goods, ideas and people between locations drives the evolution of population and productivity over time. Now it is natural to ask–what is the behavior of this system over the long run? Will population and technology continue to grow indefinitely, or will they stagnate? If the processes of population growth, migration, innovation and diffusion continue indefinitely in the absence of any changes to the transport network, where will people live, and how productive will they be?

As I will show in this section, each of these questions is liable to an analytical answer. In the long run, the economy must converge to a state in which the growth rates of population and technology are non-negative and the same in all locations, and in which all locations are populated. This state may be a Malthusian steady state in which growth rates are equal to zero, or an asymptotic balanced growth path with strictly positive growth. Which of these states comes to be depends on the overall level of transport costs, which are be summarized by a simple network statistic. In all cases, population and productivity agglomerate in central locations, according to a definition of centrality with clear roots in the network literature.

As a first step towards formalizing these statements, let us define the two possible types of long run states.

Definition 1 *A* ***Malthusian steady state*** *is a dynamic spatial equilibrium such that population* $x_i(t) = x_i$ *and idea stocks* $m_i(t) = m_i$ *in all locations* $i \in N$ *are both constant over time.*

Definition 2 *A* ***balanced growth path*** *is a dynamic spatial equilibrium such that population* $x_i(t) = (1+g_x)^t x_i$ *and manufacturing potential* $m_i(t) = (1+g_m)^t m_i$ *in all locations* $i \in N$ *grow at constant instantaneous rates* $g_x > 0$ *and* $g_m > 0$*, respectively.*

3.1 Real income under balanced growth

Labor is a key ingredient in innovation, and innovation drives non-agricultural productivity. Therefore it is no surprise that in the long run technology levels, as well as levels of real income, are a function of the distribution of population. If the growth rate of technology is constant, then the long run idea stock in location i is given by the following:

$$\begin{aligned} m_i(t)^{\frac{1}{\psi}} &= \sum_{j\in N}\sum_{s=0}^{\infty}(1-\delta)^s\theta_{ji,s}\hat{m}_j(t-s) \\ &= \sum_{j\in N}\hat{m}_j(t)\sum_{s=0}^{\infty}\left(\frac{1-\delta}{1+g_m}\right)^s\theta_{ji,s} \\ &= B_{m,2}\sum_{j\in N}\tilde{\theta}_{ji}^{\{g_m\}}x_j(t)^{\eta\phi}\lambda_j^{1-\eta\phi}, \end{aligned}$$

where $\tilde{\theta}_{ji}^{\{g_m\}} \equiv \frac{\delta+g_m}{1+g_m}\sum_{s=0}^{\infty}\left(\frac{1-\delta}{1+g_m}\right)^s\theta_{ji,s}$, and $B_{m,2} \equiv \frac{1+g_m}{\delta+g_m}B_m$. Note that the definition of $\tilde{\theta}_{ji}^{\{g_m\}}$ implies that it takes values only between zero and 1, as $\tilde{\theta}_{ii}^{\{g_m\}} = \frac{\delta+g_m}{1+g_m}\sum_{s=0}^{\infty}\left(\frac{1-\delta}{1+g_m}\right)^s = 1$ for $\forall i \in N$.

From this expression we can infer that if the amount of available labor (population) does not grow, technology will not grow, either. We can also infer that under balanced growth it must hold that $1+g_m = (1+g_x)^{\eta\phi\psi}$, and that

$$m_i^{\frac{1}{\psi}} = B_{m,2}\sum_{j\in N}\tilde{\theta}_{ji}^{\{g_m\}}x_j^{\eta\phi}\lambda_j^{1-\eta\phi} \tag{22}$$

Incorporating this information into a calculation of real income, we can apply equation (22) to (11) and (12) and write the following expression:

$$\begin{aligned} &u_i\frac{(1+g_u)^t}{(1+g_x)^{\frac{\chi\phi\psi\nu_1}{\nu_2}t}} = \\ &= B_{u,2}\left(\frac{\lambda_i}{x_i}\right)^{\nu_2}\left\{\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}\left[\frac{A\alpha_j^{\chi}}{(1+g_x)^{\eta\phi\psi t}}+(1-A)\left(B_{m,2}\sum_{k\in N}\tilde{\theta}_{kj}^{\{g_m\}}x_k^{\eta\phi}\lambda_k^{1-\eta\phi}\right)^{\chi\psi}\right]\right\}^{\nu_1}, \end{aligned}$$

where $B_{u,2} \equiv B_u\Gamma\left(1-\frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}}$ and g_u is defined as the constant growth rate of real income. From this expression we can infer that under balanced growth, $1+g_u = (1+g_m)^{\chi\psi\nu_1}$ must hold. We can also infer that that if growth rates are strictly positive, the contribution of agricultural potential α_i for $i \in N$ to real income approaches zero. Therefore, in a steady state, real income is given by the following expression:

$$u_i = B_{u,2}\left(\frac{\lambda_i}{x_i}\right)^{\nu_2}\left\{\sum_{j\in N}\left(\frac{P_i}{P_j}\right)^{\chi}\gamma_{ji}^{\chi}\left[A\alpha_j^{\chi}+(1-A)\left(B_{m,2}\sum_{k\in N}\tilde{\theta}_{kj}^{\{g_m\}}x_k^{\eta\phi}\lambda_k^{1-\eta\phi}\right)^{\chi\psi}\right]\right\}^{\nu_1}. \tag{23}$$

In a balanced growth path with $g_m > 0$, real income is given by the following expression:

$$u_i = B_{u,3} \left(\frac{\lambda_i}{x_i}\right)^{\nu_2} \left\{ \sum_{j \in N} \left(\frac{P_i}{P_j}\right)^{\chi} \gamma_{ji}^{\chi} \left(\sum_{k \in N} \tilde{\theta}_{kj}^{\{g_m\}} x_k^{\eta\phi} \lambda_k^{1-\eta\phi} \right)^{\chi\psi} \right\}^{\nu_1}, \tag{24}$$

where $B_{u,3} \equiv B_{u,2}(1-A)^{\nu_2} B_{m,2}^{\chi\psi\nu_1}$.

The interpretation of these expressions is straightforward. In the long run, real income in each location depends first of all on the number of people living in that location (x_i), and the amount of land available to divide between them for use in housing and production (λ_i). Second of all, it depends on the number of people living in every other location (x_k), because a certain portion of those people are working every period to come up with new productivity-enhancing ideas. The ideas that are discovered in each location k accumulate not only in i, but also in each of location i's trading partners, at certain rates ($\tilde{\theta}_{kj}^{\{g_m\}}$). The resulting stocks of ideas in each of these trading partners j, along with the transport cost γ_{ji} and the equilibrium ratio of the costs of production $\frac{P_i}{P_j}$, determines the contribution of this trading partner to location i's real income.

If the world is in a Malthusian steady state, trade access to agriculturally fertile locations also contributes to real income. If the world is on a positive growth path, the contribution of agriculture in the long run is negligible relative to that of the non-agricultural sector.

3.2 The Network of Utility Spillovers

I will now show how the distributions under each of these long-run configurations, and the conditions for convergence to each of them, can be characterized in terms of a network of utility spillovers. For this end, it is convenient to state the system of equations represented by (23) and (24) using matrix notation. All of the analysis that follows will be conducted under the assumption that $\psi = \frac{1}{\chi}$.

Let $\mathbf{I}$ represent an n-dimensional identity matrix, and let us define $\boldsymbol{\alpha}$ as the $n \times 1$ vector such that the i^{th} element is equal to α_i; $\mathbf{x}^{\{k\}}$ as the $n \times 1$ vector such that the i^{th} element is equal to x_i^k; $\boldsymbol{\Lambda}$ as the $n \times n$ diagonal matrix such that the ii^{th} element is equal to λ_i; $\boldsymbol{\Xi}$ as the $n \times n$ diagonal matrix such that the ii^{th} element is equal to ξ_i; $\mathbf{G}$ as the $n \times n$ matrix whose ij^{th} element is equal to $\left(\frac{P_i}{P_j}\gamma_{ij}\right)^{\chi}$; $\boldsymbol{\Theta}^{\{\varsigma_m\}}$ as the $n \times n$ matrix such that the ij^{th} element is equal to $\tilde{\theta}_{ij}(\varsigma_m)$; and $\mathbf{U}$ as the $n \times n$ diagonal matrix such that the ii^{th} element is equal to u_i.

It is also convenient to define $\bar{u} \equiv \max_{i \in N} u_i$, and $\tilde{u}_i \equiv \frac{u_i}{\bar{u}}$, and $\tilde{\mathbf{U}}$ as the $n \times n$ diagonal matrix such that the ii^{th} element equals $\tilde{u}_i$, so that $\mathbf{U} = \bar{u}\tilde{\mathbf{U}}$.

Now, (23) can be stated as

$$\bar{u}^{\frac{1}{\nu_1}}\tilde{\mathbf{U}}^{\frac{1}{\nu_1}}\mathbf{x}^{\{\frac{\nu_2}{\nu_1}\}} = A\psi_{u,2}^{\frac{1}{\nu_1}}\mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}}\mathbf{G}'\boldsymbol{\alpha} + (1-A)\psi_{u,2}^{\frac{1}{\nu_1}}\psi_{m,2}\mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}}\mathbf{G}'\mathbf{\Theta}^{\{0\}\prime}\mathbf{\Xi}\mathbf{\Lambda}^{(1-\eta)\phi}\mathbf{x}^{\{\eta\phi\}}$$

and (24) can be stated as

$$\bar{u}^{\frac{1}{\nu_1}}\tilde{\mathbf{U}}^{\frac{1}{\nu_1}}\mathbf{x}^{\{\frac{\nu_2}{\nu_1}\}} = \psi_{u,3}\mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}}\mathbf{G}'\mathbf{\Theta}^{\{\varsigma_m\}\prime}\mathbf{\Xi}\mathbf{\Lambda}^{(1-\eta)\phi}\mathbf{x}^{\{\eta\phi\}}$$

There emerges from both of the above equations a key matrix, $\mathbf{\Omega}$, which may be thought of as the adjacency matrix of the network a utility spillovers:

$$\mathbf{\Omega} \equiv \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}}\mathbf{G}'\mathbf{\Theta}^{\{0\}\prime}\mathbf{\Xi}\mathbf{\Lambda}^{(1-\eta)\phi}.$$

The ij^{th} element of this matrix is equal to

$$\omega_{ij} = \lambda_i^{\frac{\nu_2}{\nu_1}}\left[\sum_{k\in N} g_{ki}\tilde{\theta}_{jk}(0)\right]\xi_j\lambda_j^{(1-\eta)\phi},$$

and represents the extent to which consumers in location j contribute to the utility of consumers in location i in the long run. This depends, naturally, on the amount of land available for productive use in location j, and on the product, for each location $k \in N$, of location j's contribution to that location's technology level through diffusion, and location i's trade connection to that location. In other words, the benefit that location i gets from population in location j depends on technology spillovers from j, not only to i directly, but also to each of i's trading partners.

Let the largest eigenvalue of this matrix be denoted π. π is a natural statistic to summarize the world's long-run *global potential.* Not surprisingly, this productive potential is strictly increasing in the land endowment of each location, λ_i, and strictly decreasing in the bilateral transport cost between each pair of locations, $\frac{1}{\gamma_{ij}}$. As we will see in the theorem that follows, the level of π is crucial to determining whether the world stagnates or achieves sustained growth.

3.3 Conditions leading to stagnation or sustained growth

Theorem 1 *In the environment that has been described, given a vector of starting conditions* $\mathbf{s} \in \mathbb{R}_+^{n^3(n-1)}$ *containing population* x_i *for* $i \in N$*, locally-invented ideas* $m_{i,I}$ *for* $i \in N$*, and diffused ideas* $m_{ij,D}$ *for* $i \in N$*,* $j \neq i$*, such that population* x_i *in at least one location is strictly positive:*

A. If $\frac{\nu_2}{\nu_1} > \eta\phi$*, the world will converge to a unique Malthusian steady state in which every location* $i \in N$ *has positive population.*

B. If $\frac{\nu_2}{\nu_1} = \eta\phi$, then there exists a critical level of global productive potential π^*, such that

i. if $\pi \leq \pi^*$, the world will converge to a unique Malthusian steady state with positive population in each location $i \in N$, and

ii. if $\pi > \pi^*$, the world will asymptotically approximate a unique balanced growth path with positive population in each location $i \in N$.

C. If $\frac{\nu_2}{\nu_1} < \eta\phi$, then there exists a critical level of global productive potential π^* and a critical frontier of starting conditions defined by a continuous, increasing function $z(.)$ mapping from $\mathbb{R}_+^{n^3(n-1)}$ to $\mathbb{R}$, such that

i. if $\pi \leq \pi^*$ and $z(\mathbf{s}) \leq 0$, then the world will converge to a Malthusian steady state with positive population in each location $i \in N$, which may or may not be unique,

ii. if either $\pi > \pi^*$ or $z(\mathbf{s}) > 0$, the world will asymptotically approximate a balanced growth path with positive population in each location $i \in N$, which may or may not be unique.

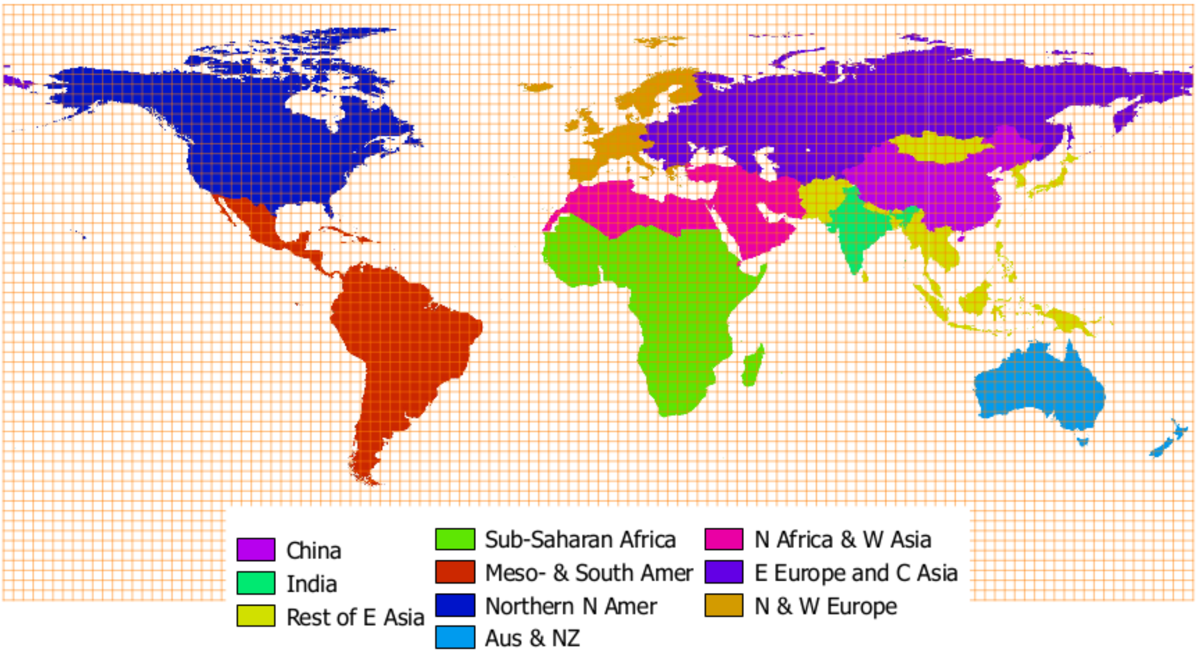
Proof: See Appendix A.6.

What Theorem 1 states is that (a) if dispersion forces are stronger than agglomeration forces, sustained growth is not possible in the long run, (b) if the forces of agglomeration are equally balanced with the forces of dispersion, then sustained growth will occur as long as global productive potential is high enough, and (c) if agglomeration forces are stronger than dispersion forces, then sustained growth will occur if either global productive potential is high enough, or if starting levels of population and technology are high enough. This suggests at least three types of change, exogenous to the model developed here, that could push a system which is in a Malthusian steady state into a path towards sustained growth:

1. a major technological breakthrough that raises the level of technology in a discrete jump, leading to an increase in $z(\mathbf{s})$
2. the creation of additional land, leading to an increase in π
3. a reduction in transport costs, leading to an increase in π

So, if technological progress is essentially incremental, and if land-recovery efforts like those undertaken in the Netherlands are not a major force in economic growth globally, this leaves option number three. Indeed, a fall in transportation costs is one of the key economic facts of the past several centuries. Interpreted through the lens of the framework developed here, then, these reductions may have been a necessary condition for the take-off in global growth that has occurred.

Figure 1: Major regions and 3° resolution grid



4 Bringing the Model to the Data

To bring the model to the data, I divide the world into 3° × 3° quadrangles. I discard all quadrangles that do not contain land, and all of the quadrangles in Antarctica. This leaves 2,249 habitable locations. Figure 1 shows the 3 degree grid. It also shows the extents of 10 major regions, which play no role in the model or its computation, but are used to aggregate results up for comparison.

4.1 Agricultural potential from agricultural characteristics

I assign agricultural potential to each location based on the index of agricultural suitability provided by Ramankutty et al. (2002). To ensure that the index I use reflects only exogenous climate and soil characteristics which are stable over time, I regress the Ramankutty index on three variables which arguably do have these properties, and use the predicted values as my index of agricultural potential.

These three variables are the Normalized Difference Vegetation Index (NDVI), soil nutrient availability, and soil workability. NDVI is a measure of how "green" a location is when observed from a satellite.[5] This measure captures how favorable are basic climatic conditions, such as water availability and temperature, for the growth of vegetation.[6] I

[5]Monthly observations for NDVI from February 2000 through January 2016 were taken from NASA LP DAAC (2016). The measure analyzed is the mean NDVI for each location over this entire time period.

[6]An alternative measure of water availability would be average rainfall. This measure has one key drawback, however: it cannot account for the lushness of certain river valleys, such as the Nile river delta, which in spite of having very little rainfall, are very "green," highly productive agriculturally, and

use indexes of soil nutrient availability and soil workability calculated by Fischer, et al (2008) for the United Nations Food and Agriculture Organization.

I specify a log-log relationship between these variables and the Ramankutty index, with a quartic polynomial in NDVI, quadratic terms for each of the soil quality measures, and a full set of interaction terms. Let the predicted values resulting from this projection be designated $\hat{a}_i$. Agricultural potential is then assigned according to

$$\alpha_i = \zeta_a \hat{a}_i,$$

where $\zeta_a > 0$ is a scale parameter which is calibrated to target the agricultural labor share in Europe in 1000 CE.

4.2 Transport costs from topography

I take information on the location of land, lakes, rivers and coastlines from the Natural Earth database. Navigable rivers are classified as those with a scalerank of 5 or lower in the Natural Earth data set, and this set is further pared by researching the navigability of the individual river systems that remain using a variety of sources, mimicking the methodology of Henderson et al (2016). I use Nunn and Puga's (2012) calculations of the Terrain Ruggedness Index proposed by Riley, DeGloria, and Elliot (1999). I use mean wave height calculations from Barstow, et al (2009).

Transportation costs between each pair of habitable locations may be carried out using land transport, river transport, sea transport, or a combination of all three. Transport is modeled as taking place on a network in which there are land, river and sea nodes. In each grid square, there exists one land node for each disjoint body of land which is at least partly inside the square, one river node for each navigable river system which is at least partly inside the square, and one sea node for each disjoint body of water that is at least partially inside the square. Each land node is directly connected to any land nodes in the eight adjacent grid squares which belong to the same body of land, and any river or sea nodes in the same grid square. Similarly, each river and sea node is directly connected to any river node, or sea node, respectively, in the eight adjacent grid squares, and any sea or land node, or river or land node, in the same grid square.

Land-land and sea-sea connections between two grid squares i and j, $i \neq j$, each face a mode-specific per-unit effective distance, $\tau_L(t)$ or $\tau_S(t)$ respectively, which is multiplied by the great circle distance d_{ij} between the centers (centroids) of the two grid squares (latitude-longitude quadrangles) to obtain the effective distance between the two nodes.[7] River-river connections face a per-unit effective distance of $\tau_S(1+\tau_V)$, where τ_V represents the increased cost which may be incurred due to the special difficulties of river navigation,

very densely populated.

[7] All distances are calculated taking the curvature of the Earth into account.

relative to navigation on calm seas. Let the arc between the centers of squares i and j be divided into two segments, one, of length d^i_{ij}, running from the center of i to the border between the squares, and a second of length d^j_{ij}, running from the center of j to the border between the squares.[8] The effective distance of land-land connections is also multiplied by $1 + \tau_R \frac{d^i_{ij}\mathfrak{r}_i + d^j_{ij}\mathfrak{r}_j}{d_{ij}}$, where $\mathfrak{r}_i$ and and $\mathfrak{r}_j$ represent the average ruggedness of the terrain in grid squares i and j, respectively.[9] The effective distance of water-water connections $1 + \tau_W \frac{d^i_{ij}\mathfrak{r}^w_i + d^j_{ij}\mathfrak{r}^w_j}{d_{ij}}$, where $\mathfrak{r}^w_i$ and $\mathfrak{r}^w_j$ are indicator functions taking a value of 1 if the seas are "rough" in square i or j, respectively, and 0 otherwise. Seas are defined as being "rough" in a given square if mean significant wave heights in that square are greater than 1.5 meters.[10] The effective distance of land-river and land-sea connections, in either direction, is equal to the transshipment cost τ_T.

If a grid square has one land node, then the effective distances faced by that land node are those also faced by the habitable location in that grid square. If there is more than one land node in a grid square, the effective distances faced by the land node are equal to the arithmetic means of the effective distances faced by the various nodes.

The effective distance between each pair of habitable locations i and j, $\tau_{ij}(t)$, is then equal to the least-cost path between them through the network.[11] The inverse iceberg transport cost γ_{ij} is then given by $\gamma_{ij}(t) = e^{\tau_{ij}(t)}$, following Allen and Arkolakis (2014).

Given initial levels $\tau_L(0)$ and $\tau_S(0)$, let the basic cost of transport over land and water fall at constant rates ς_L and ς_S, such that

$$\tau_k(t) = (1 - \varsigma_k)^t \tau_k(0) \tag{25}$$

for $k \in \{L, S\}$ and $t \in \{0, 1, 2, ..., T\}$.

4.3 Net fertility

It is assumed that annual log net fertility is related to real GDP per capita, $\tilde{u}_i$, according to the following relation:

$$\log \tilde{f}_i(\tilde{u}_i) = \left\{ \left(1 + e^{\zeta_{f,0} + \zeta_{f,1}\tilde{u}_i}\right)^{-1} \zeta_{f,2} + \left[1 - \left(1 + e^{\zeta_{f,0} + \zeta_{f,1}\tilde{u}_i}\right)^{-1}\right] \zeta_{f,3} \right. \\ \left. -2\left(1 + e^{\zeta_{f,4}\tilde{u}_i}\right)^{-1} (.5 - \zeta_{f,5}) - \zeta_{f,5} \right\}. \tag{26}$$

[8]Due to the curvature of the globe, these two segments will never be exactly equal in length, as they would be if they were connecting centroids of true squares on a plane. Also note that it is a property of the longitude-latitude quadrangle grid that the arc between the centroids of two quadrangles that are adjacent diagonally will always pass through the point where the two corners of the quadrangles meet; so the arc is contained completely within the two quadrangles and does not pass through a third.

[9]I use Nunn and Puga's (2012) calculations of the Terrain Ruggedness Index proposed by Riley, DeGloria, and Elliot (1999).

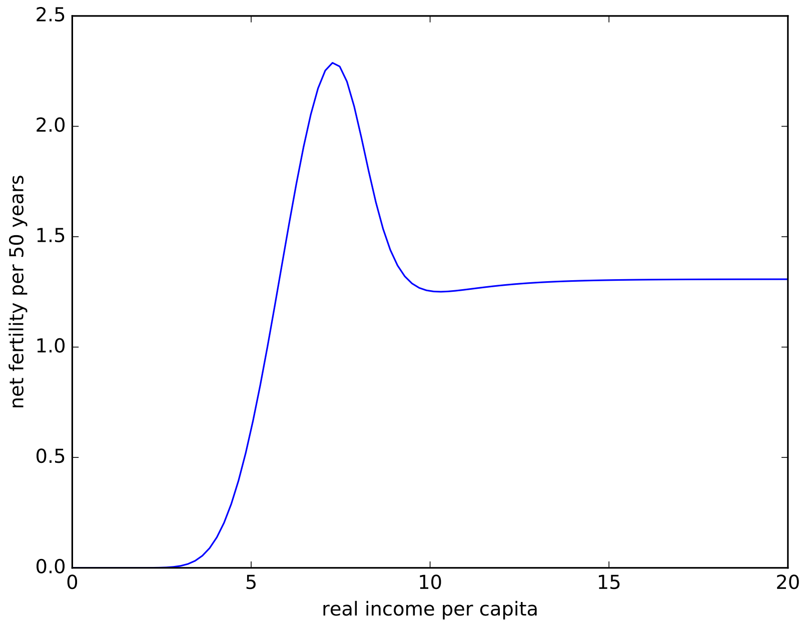
[10]I use mean wave height calculations from Barstow, et al (2009).

[11]I calculate least-cost paths using SciPy's highly-optimized implementation of Dijkstra's algorithm.

Table 1: Estimated Parameters for Fertility Process

$\zeta_{f,0}$	$\zeta_{f,1}$	$\zeta_{f,2}$	$\zeta_{f,3}$	$\zeta_{f,4}$	$\zeta_{f,5}$
-15.71	1.91	.03726	.01357	0.64	.00821

Figure 2: Net fertility as a function of real income



Real GDP per capita is assumed to correspond to utility according to $\tilde{u}_i = \tilde{\zeta}_f u_i$, where $\tilde{\zeta}_f \geq 0$ is a scalar multiplier. Parameters $\zeta_{f,k}$ for $k \in \{0, 1, ..., 5\}$ are estimated using data on rates of natural increase (birth rates minus death rates) and real GDP per capita borrowed from Delventhal, Fernández-Villaverde and Guner (2017). Table 1 shows the estimated parameters. Log net fertility per 50-year model period is then obtained by multiplying $\log \tilde{f}_i$ by 50. Figure 2 graphs the resulting function.

5 The world in 1000 CE

I conduct a quantitative exercise in two steps. First, I calibrate the model so that in the year 1000 CE the world is in a Malthusian steady state. Then, I reduce transport costs according to a pattern consistent with the existing historical evidence, and track the endogenous evolution of population and income per capita in 50-year periods until 2000 CE.

5.1 Calibration

Tables 2, 3 and 4 provide an overview of the values I choose for the parameters of the model and why. Some are set based on evidence provided by previous estimations or historical studies, and others are calibrated so that a moment of the model will exactly match a specific target which is independent of model outcomes. A small number of parameters are not tied down in either of these ways, and are set to achieve a better overall fit with the 2,249 population density moments of the initial steady state, or to achieve a better fit with qualitative features of the transition until 2000 CE.

All of the parameters that have the biggest impact on the fit of the model with the distribution of population density in 1000 CE are tied down by one of the first two methods. Those that remain to target this distribution explicitly are of secondary importance. I will now discuss each of these parameters in turn. The scale parameter on real income $\bar{\zeta}_f$, shown in Table 3, determines how real income in the model translates into fertility. In principle it would be possible to calibrate this parameter so that the model matched total world population in 1000 CE perfectly. The exact level of total world population in 1000 CE is, however, not known with great precision, so it makes more sense to allow this parameter to minimize the sum of squared errors between the model distribution of population density and the data.

The remaining parameters in this group are all initial transport cost parameters, shown in Table 4. The New World penalty, τ_{NW}, does not have a great impact on the Old World distribution of population, but it does improve model fit overall by reducing overall population density in the New World to close to the historical pre-Columbian levels. As discussed in the previous section, all indications are that New World regions lacked important transport technologies such as pack animals and sailing ships that were available throughout the Old World at this time. The penalty on rough and open seas seems to be rather well-identified, improving fit significantly when a high value is assigned to it. The penalties for traveling over rough terrain or over permafrost seem to be relatively weakly identified, though they do improve overall fit slightly when they take positive, but not very large, values.

I will now discuss each of the other parameters in the initial 1000 CE calibration. The first group of these parameters are shown in Table 2. I set $\alpha = 0.75$ according to evidence provided by Davis & Ortalo-Magné (2011), so that the share of income that consumers spend on housing is equal to 25%. By setting $\eta = 0.8$ and $\sigma = 0.2$, the land share in production is set to 16% and the intermediate input share is set to 20%, consistent with evidence provided by Desmet and Rappaport (2015) and Vandenbroucke (2008). Setting $\rho = 0.75$ implies an elasticity of substitution between goods of 4, consistent with the estimation of Bernard et al. (2003). Setting the elasticity of trade to distance $\chi = 6.5$ is consistent with evidence provided by Simonovska and Waugh (2014). Setting $\varphi = 0.5$

Table 2: Calibration, Technology and preferences
(Parameters taken from the literature)

Par.	Par. Value	Target	Target value/source
α	0.75	housing expenditure share equal to 25%	*Davis & Ortalo-Magné (2011)*
η	0.8	land share in production equal to 16%	*Desmet & Rappaport (2015)*
σ	0.2	intermediate input share equal to 20%	20% *Vandenbroucke (2008)*
ρ	0.75	elast. of subst. btw. goods equal to 4	4 *Bernard et al. (2003)*
χ	6.5	trade elasticity to distance	*Simonovska & Waugh (2014)*
φ	0.5	land share, housing prod.	*Albouy & Ehrlich (2017)*
κ	0.5	elast. of TFP to innov.	*Desmet & Rossi-Hansberg (2015)*

implies a land share in housing production consistent with Albouy and Ehrlich's (2017) study. My source for the value of κ, the elasticity of TFP to innovation effort, is Desmet and Rossi-Hansberg (2015). I set $\kappa = 0.5$.

The second group of this parameters is shown in Table 3. The elasticity of diffusion probability to distance, ζ_d, is calibrated so that the expected diffusion time from Baghdad to Pisa, Italy is 350 years. These two points and this length of time are chosen with reference to the diffusion of Indian numerals from the Middle East to Western Europe during the Middle Ages. In 825 CE Al-Khwarizmi, namesake of the word "algorithm", published a treatise on the use of Indian numerals. Knowledge of this method of numerical representation had recently spread to Al-Khwarizmi's city, Baghdad, from its place of origin in the Indian sub-continent.[12] In 1202 CE Fibonacci (of the "Fibonacci sequence") published his treatise *Liber Abaci*, the first known work by a Western mathematician comparing what Fibonacci now dubbed "Arabic numerals" to the Roman system of representation, and describing their use in performing calculations.[13]

The elasticity of migration to distance $\zeta_{m,1}$, is set so that in an idealized flat, homoge-

[12] Another mathematician, Al-Kindi, is known to have published a treatise on the same topic in either Baghdad or Basra in 830 CE.

[13] The very first known reference to Indian numerals in Western Europe is contained in the *Codex Vigilanus* compiled by monks in Abelda de Iregua, Spain around 976 CE. I use the timing implied by the publication of *Liber Abaci* because then the event in Baghdad and the event in Pisa are like to like: both are treatises written by a well-known mathematician fully explaining the subject. Presumably knowledge of Indian numerals also existed in the Islamic world in a more obscure way for some decades or centuries before the publication of Al-Khwarizmi's treatise.

Table 3: Calibration: Diffusion & migration

Par.	Par. Value	Target	Source
ϕ	1	normalization	–
ζ_d	41.8	1000 CE expected diffusion time from Baghdad to Pisa equal to 350 years (diffusion time of Indian numerals)	*Devlin (2011)* & *Berggren (1986)*
$\zeta_{m,1}$	31.7	% of residents in idealized steady state from $>$ 50km distant equal to 15%, as in migration in 14th C. Nottinghamshire	*Whyte (2000)*
ψ	2.63	BGP ratio of pop./income growth equal to 2, as in U.S. 1960-2010	*Maddison 2010 dataset*
ζ_a	8.87	1000 CE agriculture labor share in Europe equal to 85%	*Allen (2000)*
$\bar{\zeta}_f$	.1	1000 CE pop. densities	–
$\zeta_{m,0}$	0.1	evolution of population, 1000-2000 CE	–
ω	0.3	evolution of population, 1000-2000 CE	–

Table 4: Calibration, Initial Transport Costs

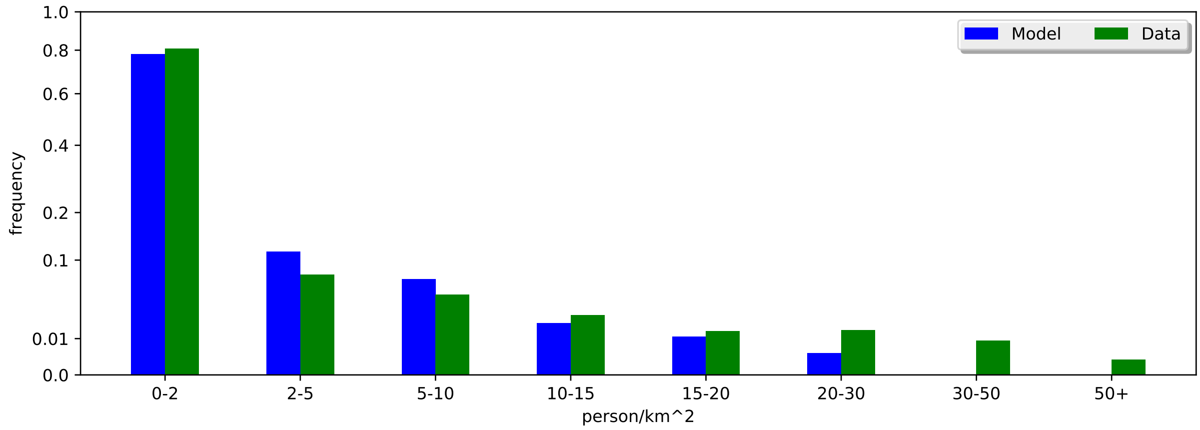
Par.	Par. Value	Target	Source
τ_L	.08	increase in wheat price of 8% per 111km in 14$^{\text{th}}$ C. Engl.	*Masschaele (1993)*
τ_S	$\frac{\tau_L}{8}$	ratio of coastal waters to land transport cost in 14$^{\text{th}}$ C. Engl.	*Masschaele (1993)*
τ_V	$2\tau_S$	ratio of river to coastal waters transport costs in 14$^{\text{th}}$ C. Engl.	*Masschaele (1993)*
τ_T	$1.47\tau_V$	ratio of transhipment cost per ton to river transport cost to move 1 ton 111 km in 19$^{\text{th}}$ U.S.	*Fogel (1962)*
τ_W	15	1000 CE pop. densities	–
τ_R	1	1000 CE pop. densities	–
τ_F	1	1000 CE pop. densities	–
τ_{NW}	24	1000 CE pop. densities	–

neous, endless plain in a steady state, the fraction of residents living at any given point who were born more than 50 kilometers away is equal to 15%. This is consistent with evidence on migration in rural 14th Century Nottinghamshire compiled by Whyte (2000). The scale parameter on agricultural potential ζ_a, is set so that the agriculture share of employment in Europe in 1000 CE is equal to 85%, consistent with evidence on Medieval European agriculture shares complied by Allen (2000). I normalize the elasticity of idea creation to the density of innovation effort ϕ, to equal 1, and set the elasticity of the effective technology level to the stock of ideas ψ equal to 2.63, which implies a balanced growth path income per capita will grow twice as fast as population. This is consistent with the data on population and income per capita growth in the United States from 1960 to 2010.

The last group of parameters is shown in Table 4. I set τ_L so that the price change per 111 kilometers (1° latitude) is 8%, which is near the middle of the range of price to distance elasticities that Masschaele (1993) finds for wheat being transported over land in 14th Century England. Masschaele (1993) also finds a ratio of the average land transport cost to the average coastal waters transport cost of 8 to 1, and an average ratio of the river transport cost to coastal waters transport cost of 2 to 1; I use these numbers as is. Fogel (1962) estimates that the cost per ton of loading or unloading goods from a boat is

Figure 3: Distribution across population density levels in 1000 CE, model and data

3° × 3° grid squares: **R^2:** .31 **weighted corr:** .57



1.47 times the cost of transporting the same ton of goods on a river for 111 kilometers; I use this number as well.

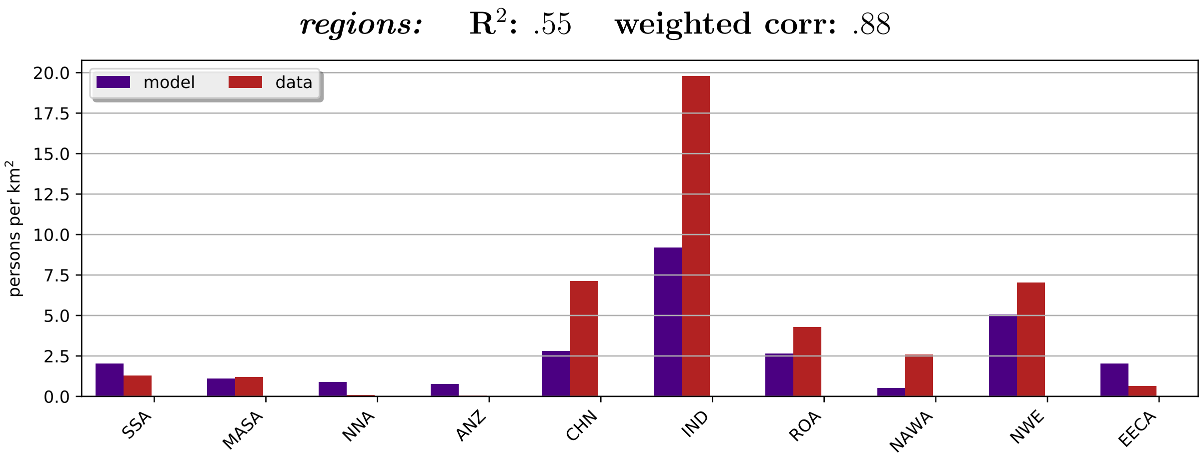
The parameters $\psi_{m,0}$ and ω, shown in Table 3, while they do have some effect on the distribution of population in 1000 CE, are set to match some qualitative features of the evolution of population between 1000 and 2000 CE and are discussed in Section 6.

5.2 Results, 1000 CE

The overall fit of the model with the data in 1000 CE is summarized in Figures 3 and 4. At the level of 3° by 3° quadrangles, the model is able to to account well for the distribution of locations across population density levels, though it is unable to generate the handful of locations with very high density which exist in the data. The model is able to do a good job accounting for which specific locations have low and high density as well, accounting for 31% of the overall variation.

As can be seen in Figure 4, the model also accounts well for which of the 10 major regions are densely and which are not as densely populated in 1000 CE. In the model, as in the data, India, China and Europe are the three most densely populated places in the world. The model is not able to quite match the same level of density as existed in India and China, in part because of its inability to generate very high density locations. Overall, it is able to account for most of the variation between these major regions–55%. The interpretation of this result is that agricultural potential and access to water transport, taken together, are able to account well for which regions were more and which were less developed in 1000 CE.

Figure 4: Mean population density of 10 major regions in 1000 CE, model and data



6 Falling Transport Costs

The second step of the quantitative exercise is to reduce transport costs according to a specified pattern and simulate the model until 2000 CE. This is done in two phases, as shown in Figure 5. First, between 1000 CE and 1500 CE, reduce water and land transport costs at constant rates, imposing a large reduction in water transport costs, and a much smaller reduction in land transport costs. Also over this period, the large penalty on traveling far from the coast or over rough seas is gradually removed. This is consistent with the broad pattern which has been found by Masschaele (1993) and others: that prior to the development of railroads, improvements in water transport were much more significant than any improvements in land transport. It is also consistent with the well-known developments in navigation technology over this period which culminated in the first cross-Atlantic voyages and the first circumnavigation of the globe.

From 1500 CE until 1750 CE there is a pause in the reduction of transport costs. Then

Figure 5: Falling transport costs

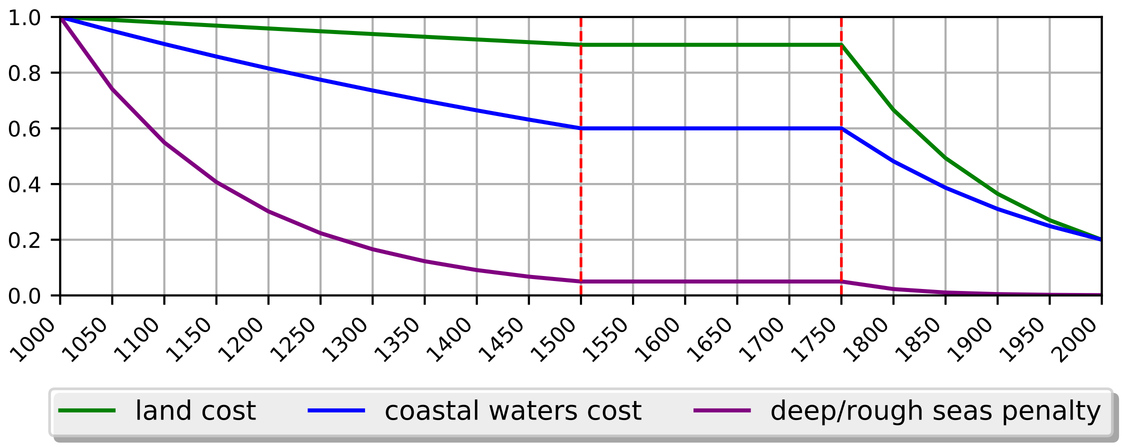


Table 5: Transport Cost Reductions

Par.	% in 1500 CE	% in 1750 CE	% in 2000 CE
τ_L	90%	90%	20%
τ_S	60%	60%	20%
τ_W	5%	5%	0.1%

from 1750 to 2000 CE transport costs are again reduced at a steady rate. This second phase of reductions is more land-biased than the first, to reflect the importance of land transport developments such as railroads and the automobile. The exact magnitudes of all of these transport cost reductions are chosen to approximate the qualitative features of the evolution of population and income between 1000 and 2000 CE. These values are shown in Table 5.

In addition to the aforementioned transport cost reductions, the penalty on transport in the New World, τ_{NW}, is removed linearly between 1450 and 1600 CE, reflecting the discovery of the Americas and Australia by Old World explorers and the spread of Old World transport technologies across the New World.

Two parameters from Table 3, $\psi_{m,0}$ and ω, are calibrated to improve the model fit with qualitative features of the evolution of population between 1000 CE and 2000 CE. $\psi_{m,0}$, which represents the inverse of the home bias exhibited by consumers in choosing migration, is chosen to ensure that a plurality of consumers stay in the locations they were born in, even in 2000 CE. ω, the elasticity of congestion to population density, is chosen to reduce the concentration of population growth in regions that take off early versus those that take off late.

6.1 Results, 1000-2000 CE

Figure 6 shows the evolution of total world population in the model and in the data. The model replicates well the overall pattern of accelerating growth in world population, with a sharp increase in growth rates after 1700 CE. The model starts with a total world population of 260 million people, which is inside the range of plausible historical estimates, and ends in 2000 CE with 6 billion, just as in the data.

As can be seen in Figure 7, the correlation between the model and the data distributions of population density, both across regions and across individual 3° by 3° locations, remains high for most of the simulation. Both of these correlations decline sharply as population growth accelerates after 1700 CE, ending in 2000 CE at lower but still positive levels.

Figure 8 compares the evolution of world mean real income per capita in the model and in the data, where the mean is taken of the natural log of real income and weighted by population. The discrepancy early in the simulation, when mean real income in the model

Figure 6: Simulation results: world population

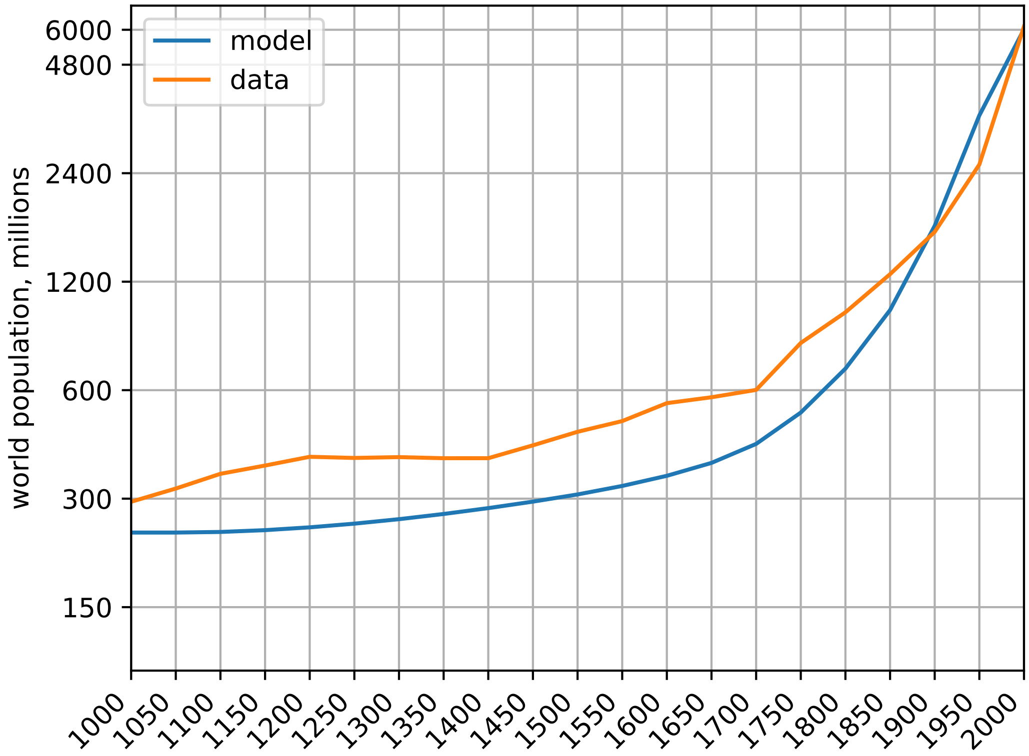


Figure 7: Simulation results: population density

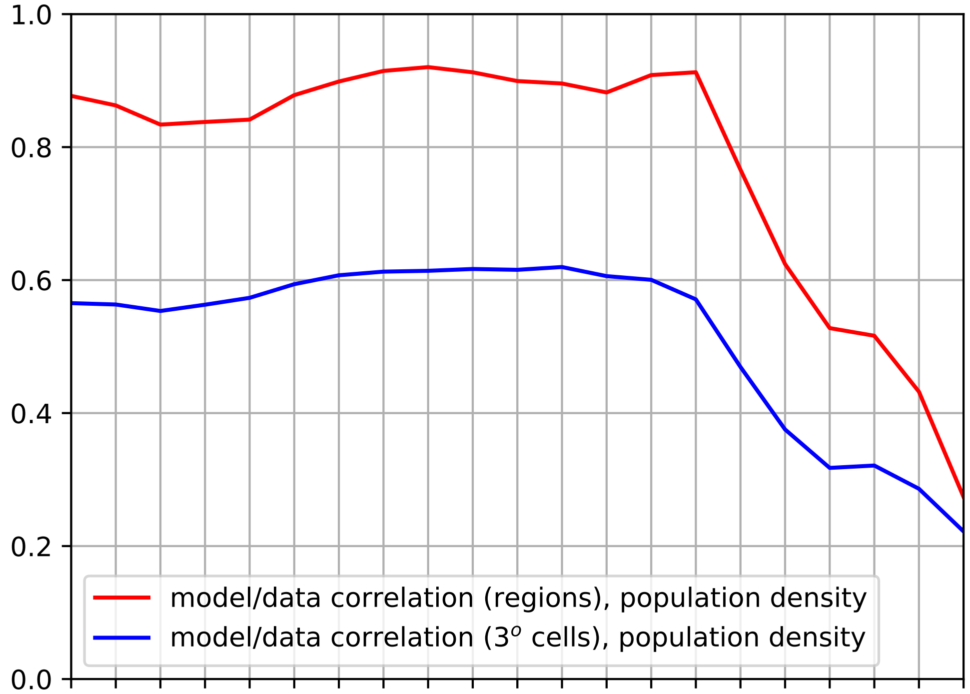
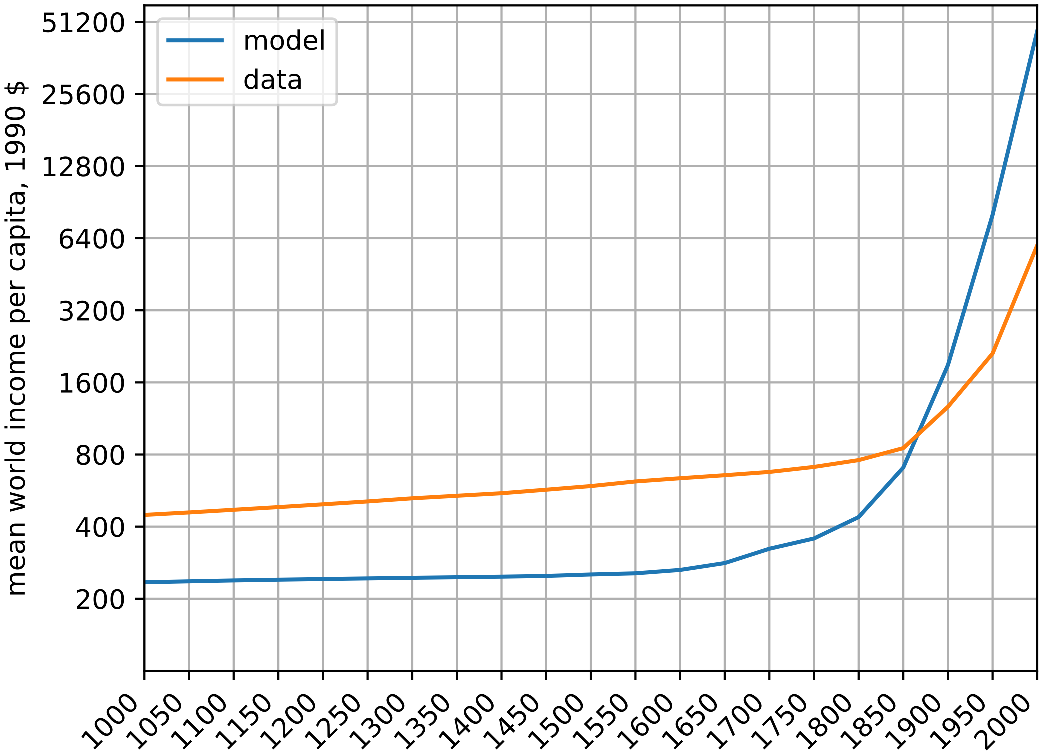


Figure 8: Simulation results: world income

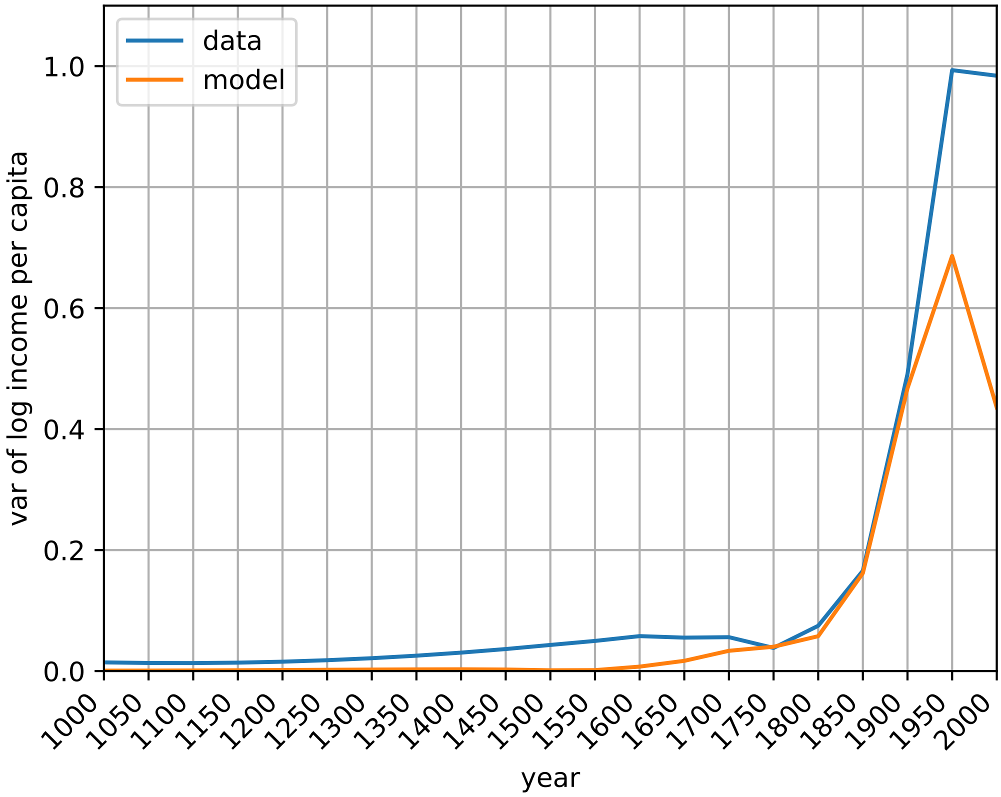


is somewhat less than that in the data, is not particularly meaningful, as the numbers for the data during this period are themselves somewhat speculative. It is clear, however, that there is much more growth in income per capita after 1800 CE in the model than in the data. Aside from this, both the model and the data display the same basic pattern of accelerating growth, which is almost flat prior to 1800 CE, and increases sharply after 1800 CE.

The model matches very well the evolution of income dispersion across regions until 1900 CE, as can be seen in Figure 9. Income dispersion is measured as the variance in log real income per capita across the 10 major regions, weighted by population. They are at the same level in 1800 CE, and move together tightly for the next 100 years. From 1900 to 1950, the increase in dispersion in the model slows down slightly, while the increase in dispersion in the model accelerates. From 1950 to 2000, dispersion declines in both the model and the data, though this decline is considerably larger in the model than in the data. In the end, the variance across regions of log income per capita in the model is 43% of what is observed in the data in 2000 CE.

The simulation also matches well the evolution of Europe's lead in income per capita over the rest of the world. Figure 10, shows the evolution of the ratio between the population-weighted mean income per capita in Europe to the population-weighted mean income per capita for the entire world. This ratio in both the simulation and the data increase steadily, though the increase is not as big in the simulation as it is in the data.

Figure 9: Evolution of income dispersion

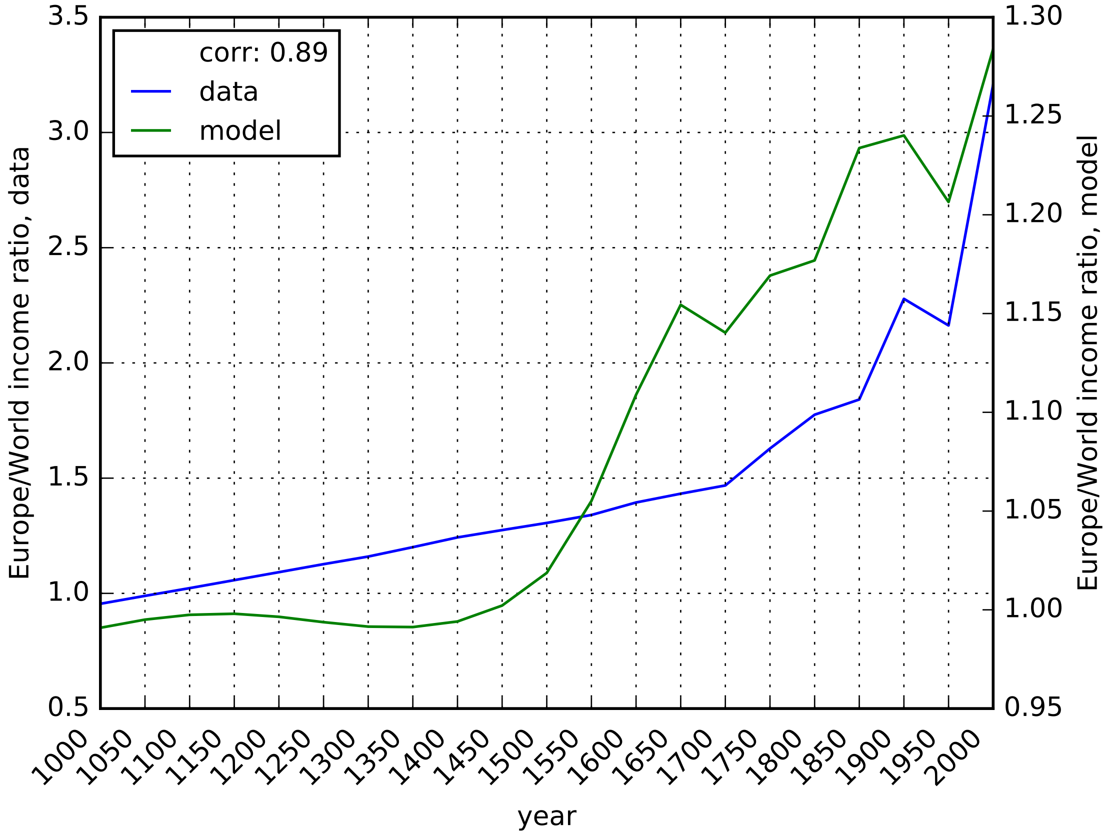


In Figure 11 we can see that the movements of this ratio in the model and in the data are highly correlated. This figure displays the evolution of the one-period growth rate of this ratio in the model and the simulation, which have a correlation of 0.51 for the entire simulation period. If we take into account the fact that the first meaningful observations for income per capita are really in 1800 CE, and so consider only the one period growth rates from 1850 onwards, this correlation is higher, at 0.66.

The first reliable data observations for income per capita begin in 1800 CE. As can be seen in Figure 12, the model at this point matches the distribution of income per capita across regions quite well. The correlation across regions between log income per capita in the model and in the data at this point is 0.66. As mean world income and the dispersion in income both increase after 1800 CE, this correlation declines.

What drives this decline in correlation? There are two main reasons: Northern North America and Australia and New Zealand do not grow enough between 1800 CE and 2000 CE, and Northern African and West Asia, Eastern Europe and Central Asia, and Sub-Saharan Africa grow too much. Figure 13 shows the correspondence between real income per capita relative to Northern and Western Europe in the data and in the model. The size of the marker for each region represents its total population. As can be seen in the figure, the distribution of income per capita across regions in the model lines up well with that in the data in 1800 CE, and the best linear fit line has a slope close to 1.

Figure 10: Europe/World Income Ratio



Looking next at Figure 14, we can see that over the intervening 200 years, we can see that the ratio between income per capita in Northern and Western Europe and in China the "Rest of Asia," Meso- and South America, and India have evolved in a manner more or less consistent with the data. Northern North America, comprising the modern countries of the United States and Canada, as well as Australia and New Zealand, however, have not grown nearly enough. Northern Africa and West Asia, and Eastern Europe and Central Asia have grown too much. And Sub-Saharan Africa has also grown too much, converging towards Europe more strongly than it does in the data.

Figure 15 compares the evolution of the correlation of log income per capita across regions between the model and the data, if the United States, Canada, Australia and New Zealand are included or excluded from the sample. We can see that excluding these four countries improves the correlation with the data considerably, especially during the 19th century. During the 20th century, however, the correlation for the reduced sample still declines steadily. One reason for this is that there is too much convergence in general in the 20th century, as we saw when analyzing the evolution of income per capita dispersion.

Figure 11: Growth Rate of Europe/World Income Ratio

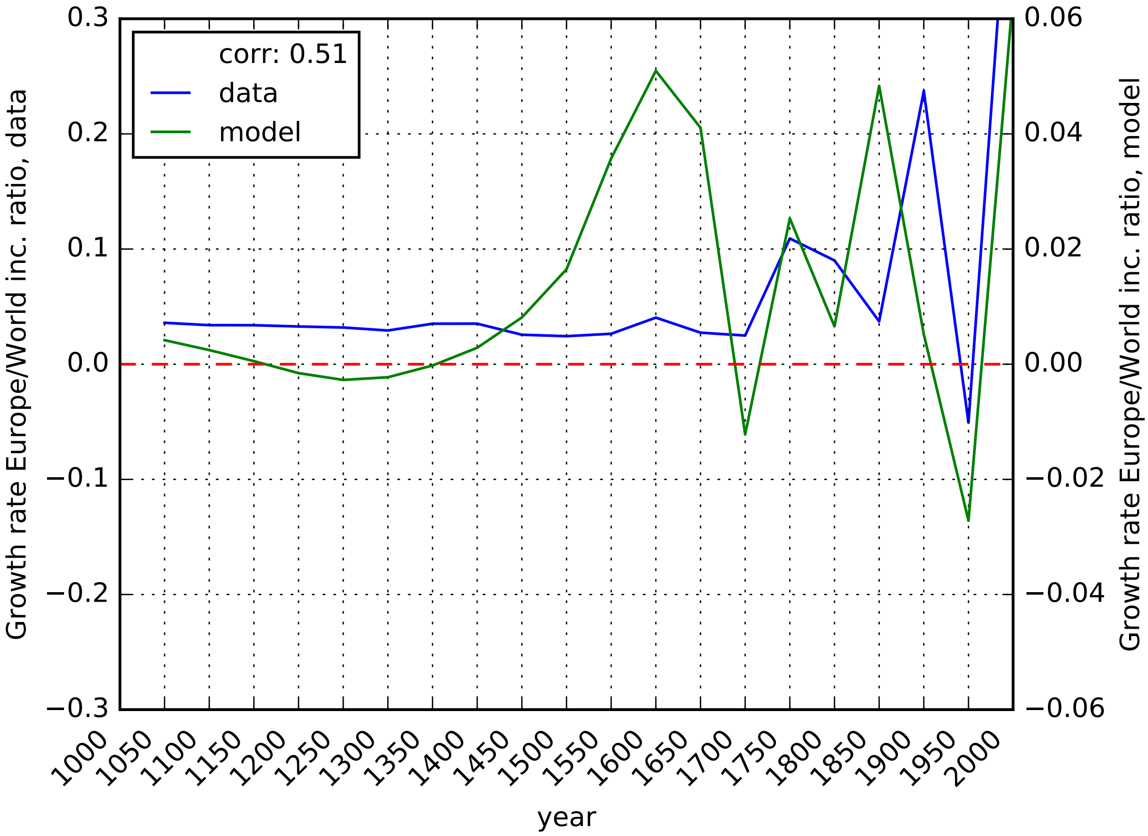


Figure 12: Simulation results: income per capita
Correlation in 1800 CE: .66

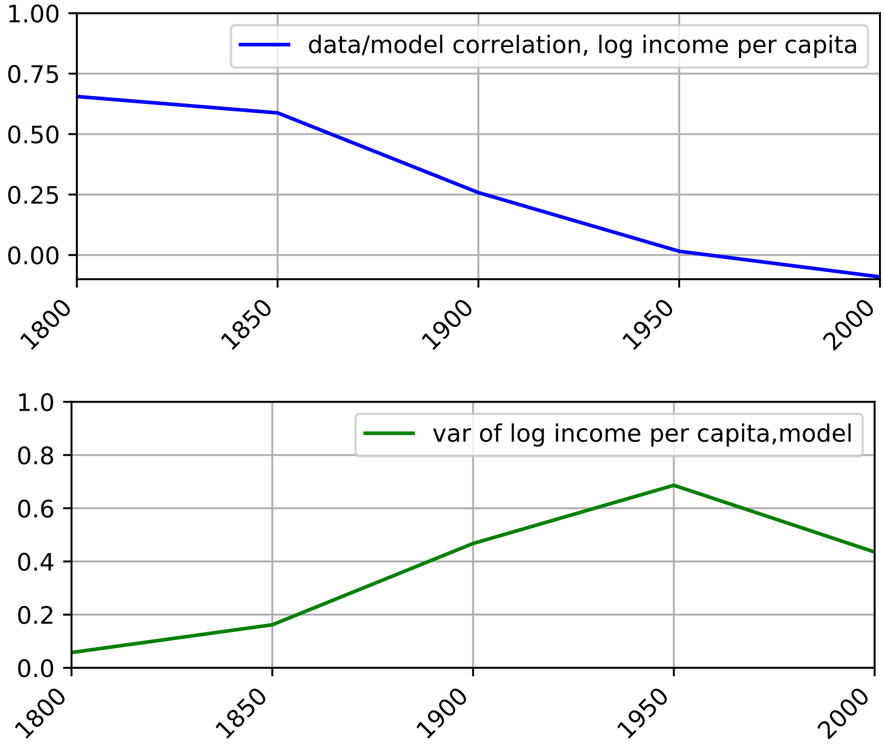


Figure 13: Income relative to Europe - 1800 CE

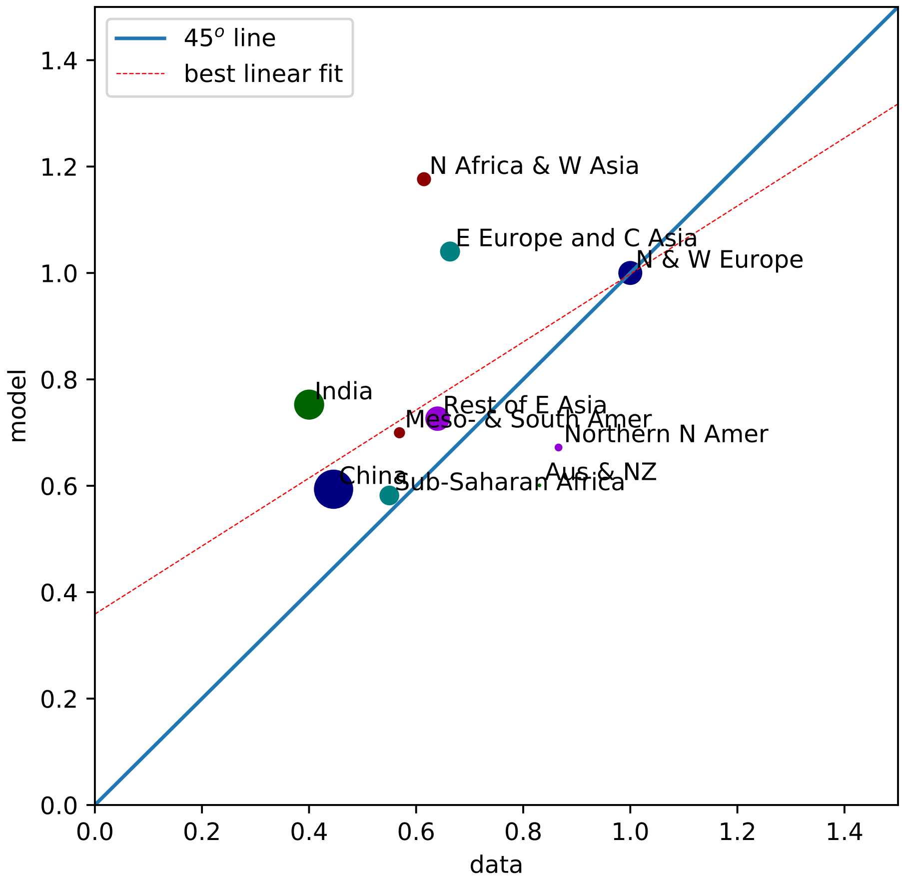


Figure 14: Income relative to Europe - 2000 CE

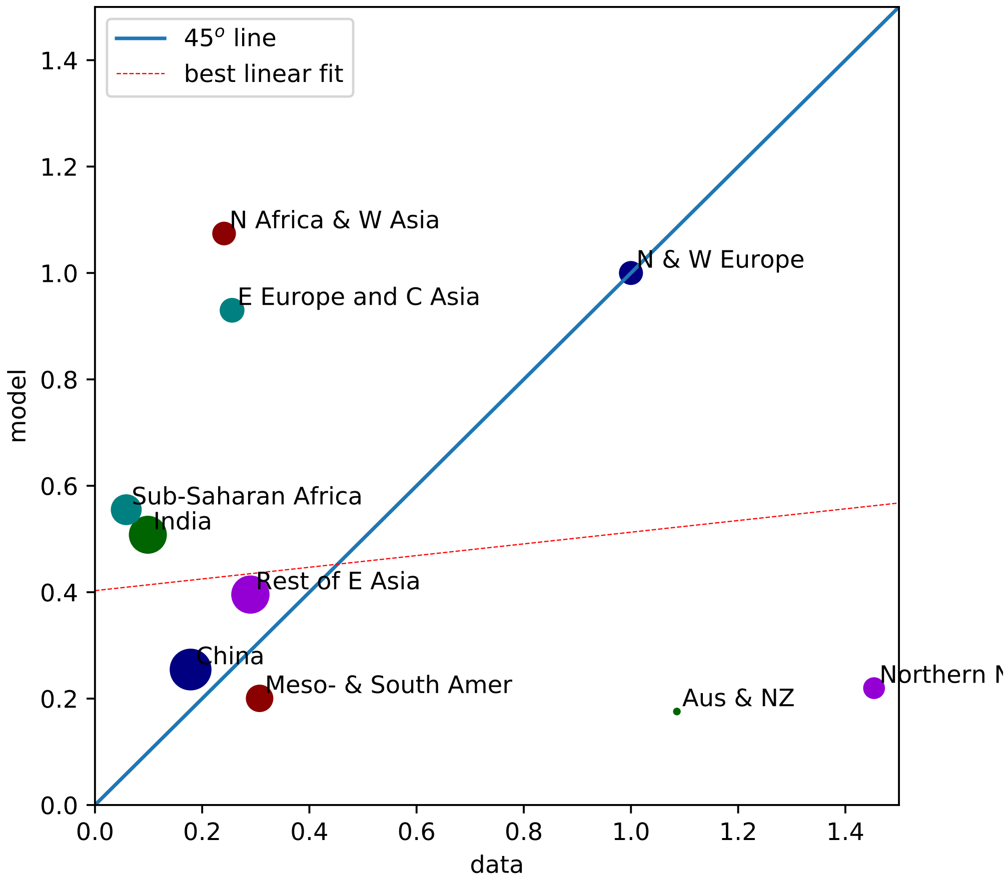
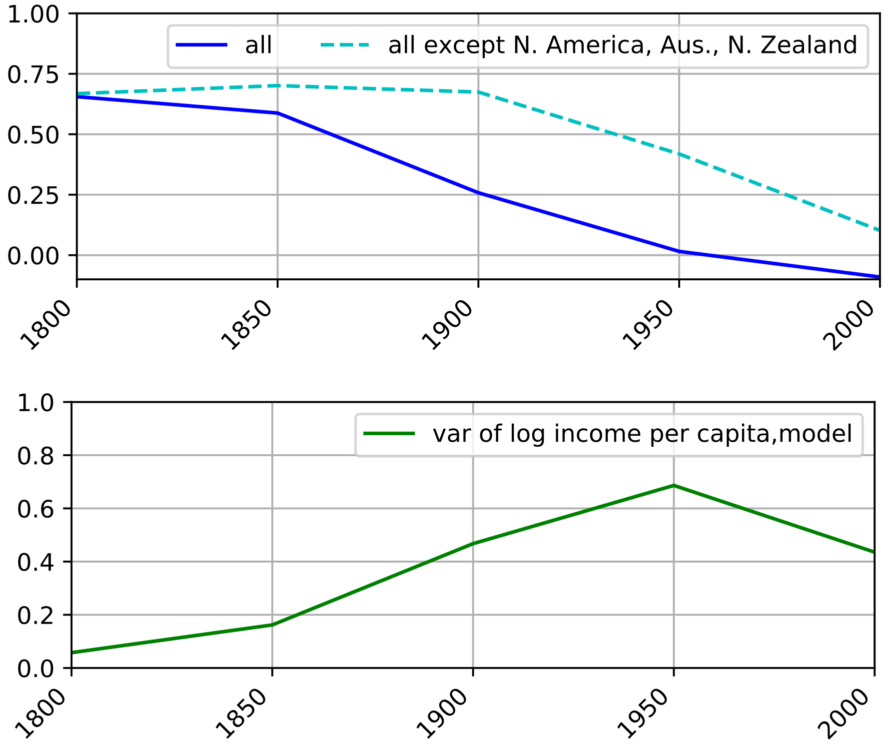


Figure 15: Simulation results: income per capita, U.S., Canada, Australia, N. Zealand excluded

Correlation in 1800 CE: .66



6.2 Discussion

What might explain the inability of the model to match the fast growth in the U.S., Canada, Australia and New Zealand after 1800 CE, and the slow convergence generally after 1900 CE? Two possible explanations in particular spring to mind. First, we know that in reality trade costs and the speed of technology diffusion depend on other factors in addition to mere transport costs. By ignoring these factors, the model effectively imposes an average trade cost level and diffusion speed for the whole world. In reality, however, it may be that the costs should be lower, and the speed higher, between Europe and the United States, Canada, Australia and New Zealand, than they are between Europe and the rest of the world. One well-known fact that might justify such a difference is that in the 19th century all these countries were populated by people speaking the same language as the leading European industrial power, England.

A second possible explanation is that there is significant variation across regions in objective institutional quality. It may be, for example, that the United States, Canada, Australia and New Zealand have better property protections or constraints than other regions, for some reason that is not directly related to access to trade or technology diffusion.[14]

[14]This hypothesis would be consistent with the findings of an extensive literature in comparative economic development, of which Acemoglu, Johnson and Robinson (2001) is a prominent example.

With this in mind, a fruitful way to extend the current exercise would be to impose additional restrictions on the model and perform counterfactual exercises to test each of these possible explanations. In this way it may be possible to determine whether either explanation is capable of reconciling the baseline model with the data, and which explanation seems to fit best.

7 Conclusion

In this paper we have seen that a pattern of falling transport costs consistent with historical evidence, applied to a spatial dynamic model in which the strength of bilateral connections is determined by the natural topography of the globe, can account for many of the important features of the evolution of the distribution of population and income over the last 1000 years. This modeling approach is able to generate initially slow, accelerating growth, with a sharp increase in population growth, income growth, and the dispersion of income across locations after1800 CE. Quantitatively, it is able to account for 55% of the variation across major regions in population density in 1000 CE, 44% of the variation across regions in income per capita in 1800 CE, and can generate 43% of the variation in income per capita across regions in 2000 CE.

This approach is also not able to match a number of facts, such as the rapid growth in income per capita in the United States, Canada, Australia and New Zealand after 1800 CE, and the slowness convergence of income per capita in the world in general during the 20th century. Future research could extend the framework presented here to test whether there are institutional or historical factors which can reconcile the model to the data on this and other points.

Another natural avenue for future research would be to try to explain the one key factor which this study has taken as exogenous–the evolution of transport costs. Why were key transport technologies developed at certain times and locations? What are the implications of allowing improvements in transport technology in some locations before others? The current framework, which is able to provide a quantitative approximation of the location-specific benefits and global aggregate consequences of transport technology changes, would be a natural starting point for such an investigation.

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A Proofs

A.1 Proof of optimal land allocation

To start with, let us state the consumer's problem:

$$\max_{\{c_{il}\}_{l\in[0,1]},h_i} \left\{ \left(\int_0^1 c_{i,l}^{\rho} dl \right)^{\frac{\alpha}{\rho}} h_i^{1-\alpha} \right\}$$

such that $w_i + p_{i,\lambda}\frac{\lambda_i}{x_i} \geq \int_0^1 p_{i,l}c_{i,l}dl + p_{i,h}h_i$.

First order conditions with respect to consumption and housing imply the following two conditions:

$$c_{il} = \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} p_{il}^{-\frac{1}{1-\rho}},$$

implying

$$C_i = \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i} \mathbb{M}_i^{\frac{1}{\chi}},$$

and

$$h_i = (1-\alpha)\frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{p_{i,h}^{-\frac{\rho}{1-\rho}}} p_{i,h}^{-\frac{1}{1-\rho}} = (1-\alpha)\frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{p_{i,h}}.$$

The production function for a goods producer with efficiency shock $s_{i,k}$:

$$q_k = s_{i,k}\left(b_{k,I}^{\eta} l_{k,I}^{1-\eta}\right)^{\kappa} \left(b_k^{\eta} l_k^{1-\eta}\right)^{1-\sigma-\kappa} \left[\int_0^1 z_{kl}^{\rho} dl\right]^{\frac{\sigma}{\rho}}$$

The cost-minimization problem of a location-i goods producer is given by

$$\min_{b_k,l_k,b_{k,I},l_{k,I},\{z_{kl}\}_{l\in[0,1]}} \left\{ w_k\left(b_k + b_{k,I}\right) + p_{i,\lambda}\left(l_k + l_{k,I}\right) + \int_0^1 p_{kl}z_{kl}dl \right\}$$

It is straightforward to solve for the optimal allocation of labor and land between improvement and production, and write the following simplified problem in terms of production land and production labor only:

$$\min_{b_k,l_k,\{z_{kl}\}_{l\in[0,1]}} \left\{ \frac{1-\sigma}{1-\sigma-\kappa}\left[w_k b_k + p_{i,\lambda}l_k\right] + \int_0^1 p_{kl}z_{kl}dl \right\}$$

such that

$$q_k = s_{i,k}\left(\frac{\kappa}{1-\sigma-\kappa}\right)^{\kappa}\left(b_k^{\eta}l_k^{1-\eta}\right)^{1-\sigma}\left[\int_0^1 z_{kl}^{\rho}dl\right]^{\frac{\sigma}{\rho}}$$

First order conditions with respect to each type of input, land, intermediate inputs and labor, imply the following two conditions relating land and intermediate good inputs to the quantity of labor input:

$$l_k = \frac{1-\eta}{\eta}\frac{w_i}{p_{i,\lambda}}b_k$$

$$z_{kl} = \frac{\sigma}{\eta(1-\sigma-\kappa)}\frac{w_i}{p_{il}^{\frac{1}{1-\rho}}P_i^{-\frac{\rho}{1-\rho}}\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}b_k$$

These then imply the following relationship between the quantity of labor input and the quantity produced:

$$q_k = s_{i,k}b_k(1-\eta)^{(1-\eta)(1-\sigma)}\left(\frac{1}{\eta}\right)^{(1-\eta)(1-\sigma)+\sigma}\kappa^{\kappa}\sigma^{\sigma}\left(\frac{1}{1-\sigma-\kappa}\right)^{\kappa+\sigma}w_i^{(1-\eta)(1-\sigma)+\sigma}\left(\frac{1}{p_{i,\lambda}}\right)^{(1-\eta)(1-\sigma)}P_i^{-\sigma}\mathbb{M}_i^{\frac{\sigma}{\chi}}$$

It also implies the following minimized cost of production in terms of quantity of production labor:

$$\begin{aligned}&\frac{1-\sigma}{1-\sigma-\kappa}\left[w_ib_k + b_k\frac{1-\eta}{\eta}w_i\right] + \frac{\sigma}{\eta(1-\sigma-\kappa)}w_ib_k\\ &= \frac{1}{\eta(1-\sigma-\kappa)}w_ib_k\end{aligned}$$

This then implies the following efficiency cost of producing a single unit of good in location i:

$$P_i \equiv \frac{s_{i,k}}{\eta(1-\sigma-\kappa)}w_ib_k(1) = \frac{w_i^{\eta}p_{i,\lambda}^{1-\eta}\mathbb{M}_i^{-\frac{1}{\chi}\frac{\sigma}{1-\sigma}}}{\eta^{\eta}(1-\eta)^{1-\eta}\sigma^{\frac{\sigma}{1-\sigma}}}\left(\frac{1}{(1-\sigma-\kappa)^{1-\sigma-\kappa}\kappa^{\kappa}}\right)^{\frac{1}{1-\sigma}}$$

Note that the actual cost faced by the producer is $\frac{P_i}{s_{i,k}}$.

Flipping this expression around, we find that

$$\frac{w_i^{\eta}p_{i,\lambda}^{1-\eta}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}} = P_i\eta^{\eta}(1-\eta)^{1-\eta}\sigma^{\frac{\sigma}{1-\sigma}}\left(\kappa^{\kappa}(1-\sigma-\kappa)^{1-\sigma-\kappa}\right)^{\frac{1}{1-\sigma}}$$

Applying this last formula to the expression for quantity produced in terms of quantity of labor employed yields the following:

$$q_k = s_{i,k} \left(\frac{w_i^{\eta} p_{i,\lambda}^{1-\eta}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}} \right)^{\sigma} w_i^{1-\eta} p_{i,\lambda}^{\eta-1} P_i^{-\sigma} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} b_k (1-\eta)^{(1-\eta)(1-\sigma)} \eta^{\eta-\eta\sigma-1} \sigma^{\sigma} \kappa^{\kappa} \left(\frac{1}{1-\sigma-\kappa} \right)^{\kappa+\sigma}$$

$$q_k = s_{i,k} b_k \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} \left(\frac{w_i}{p_{i,\lambda}} \right)^{1-\eta} \left(\frac{1-\eta}{\eta} \right)^{1-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}-1},$$

and finally:

$$q_k = s_{i,k} \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}-1} b_k^{\eta} l_k^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}.$$

Cost-minimization implies that all firms i location i must use the same ratio of land and labor, and aggregation implies that this must be equal to the aggregate ratio of land and labor used in goods production, $\frac{1-\sigma-\kappa}{1-\sigma} l_i$ and $\frac{1-\sigma-\kappa}{1-\sigma} x_i$, respectively. Then wages are given by

$$w_i = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \eta \left(\frac{l_i}{x_i} \right)^{1-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

and land rents are given by

$$p_{i,\lambda} = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} (1-\eta) \left(\frac{l_i}{x_i} \right)^{-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

Then,

$$w_i + p_{i,\lambda} \frac{\lambda_i}{x_i} = \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} \left(\frac{\lambda_i}{x_i} \right) \psi_{g,\lambda}^{-\eta} \left[\eta \psi_{g,\lambda} + 1 - \eta \right]$$

The production function of a housing producers is given by

$$H_i = \left(\int_0^1 z_{il,h}^{\rho} dl \right)^{\frac{\varphi}{\rho}} l_{i,h}^{1-\varphi}$$

The cost minimization problem of a housing producer is given by

$$\min_{\{z_{il,h}\}_0^1, l_{i,h}} \left\{ \int_0^1 p_{il} z_{il,h} dl + p_{i,\lambda} l_{i,h} \right\}$$

First order conditions imply the following relationship between quantity of intermediate input used and quantity of land used as inputs:

$$z_{il,h} = \frac{\varphi}{1-\varphi} \frac{p_{i,\lambda}}{p_{il}^{\frac{1}{1-\rho}} P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} l_{i,h}$$

Using this, housing as a function of land employed is given by

$$H_i = \left[\frac{\varphi}{1-\varphi}\frac{p_{i,\lambda}}{P_i}\mathbb{M}_i^{\frac{1}{\chi}}\right]^{\varphi} l_{i,h}.$$

This then implies the following unit cost of production for housing, which in equilibrium will also be the housing price faced by consumers:

$$p_{i,h} = \frac{1}{1-\varphi}p_{i,\lambda}l_{i,h}(1) = \frac{P_i^{\varphi}\mathbb{M}_i^{-\varphi\frac{1}{\chi}}p_{i,\lambda}^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi}}$$

Flipping this around and plugging back into the previous expression for housing in terms of land use implies

$$\frac{p_{i,\lambda}}{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}} = \varphi^{\frac{\varphi}{1-\varphi}}(1-\varphi)P_i^{-\frac{\varphi}{1-\varphi}}p_{i,h}^{\frac{1}{1-\varphi}},$$

$$H_i = \left(\frac{p_{i,\lambda}}{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}}\right)^{\varphi} P_i^{-\varphi}\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}\left(\frac{\varphi}{1-\varphi}\right)^{\varphi} l_{i,h},$$

and

$$H_i = \varphi^{\frac{\varphi}{1-\varphi}}\left(\frac{p_{i,h}}{P_i}\right)^{\frac{\varphi}{1-\varphi}}\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}l_{i,h}.$$

Then, the following relationship can be derived between price of goods and price of housing:

$$p_{i,h} = P_i\mathbb{M}_i^{\frac{\sigma-\varphi}{\chi(1-\sigma)}}\frac{\left[\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\left(\frac{l_i}{x_i}\right)^{-\eta}\right]^{1-\varphi}}{\varphi^{\varphi}(1-\varphi)^{1-\varphi}}$$

$$\left(\frac{P_i}{p_{i,h}}\right)^{\frac{1}{1-\varphi}} = \frac{\varphi^{\frac{\varphi}{1-\varphi}}(1-\varphi)}{\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}\left(\frac{l_i}{x_i}\right)^{\eta}\frac{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}}$$

Then,

$$w_i + p_{i,\lambda}\frac{\lambda_i}{x_i} = \sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}P_i\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}\left(\frac{\lambda_i}{x_i}\right)^{1-\eta}\psi_{g,\lambda}^{-\eta}\left[\eta\psi_{g,\lambda}+1-\eta\right]$$

and

$$x_ih_i = x_i(1-\alpha)\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\frac{P_i}{p_{i,h}}\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}\left(\frac{\lambda_i}{x_i}\right)^{1-\eta}\psi_{g,\lambda}^{-\eta}\left[\eta\psi_{g,\lambda}+1-\eta\right]$$

Then setting demand equal to supply, $H_i = x_i h_i$,

$$\varphi^{\frac{\varphi}{1-\varphi}} \left(\frac{p_{i,h}}{P_i}\right)^{\frac{\varphi}{1-\varphi}} \mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}} \lambda_i (1-\psi_{g,\lambda}) = x_i (1-\alpha) \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}$$

$$\frac{\varphi^{\frac{\varphi}{1-\varphi}}}{\sigma^{\frac{\sigma}{1-\sigma}}} \frac{1}{(1-\alpha)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}} \psi_{g,\lambda}^{\eta} \left(\frac{\lambda_i}{x_i}\right)^{\eta} \frac{\mathbb{M}_i^{\frac{\varphi}{1-\varphi}\frac{1}{\chi}}}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}} (1-\psi_{g,\lambda}) = \left(\frac{P_i}{p_{i,h}}\right)^{\frac{1}{1-\varphi}} [\eta\psi_{g,\lambda} + 1 - \eta]$$

$$\frac{(1-\eta)}{(1-\varphi)(1-\alpha)}(1-\psi_{g,\lambda}) = \eta\psi_{g,\lambda} + 1 - \eta$$

$$\psi_{g,\lambda}\left[\eta + (1-\eta)\frac{1}{(1-\varphi)(1-\alpha)}\right] = (1-\eta)\left[\frac{1}{(1-\varphi)(1-\alpha)} - 1\right]$$

and, finally:

$$\psi_{g,\lambda} = \frac{(1-\eta)\left(\alpha + \varphi(1-\alpha)\right)}{\eta(1-\varphi)(1-\alpha) + (1-\eta)}$$

$$= \frac{(1-\eta)\left(\alpha + \varphi(1-\alpha)\right)}{(1-\varphi)(1-\alpha) + (1-\eta)\left(\alpha + \varphi(1-\alpha)\right)}$$

A.2 Proof of equilibrium total revenue and wage

Quantity produced for a particular good in location i:

$$q_{i,k} = s_{i,k}\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}-1} b_{i,k}^{\eta} l_{i,k}^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

Revenue per unit of output for good k is given by $\frac{P_i}{s_{i,k}}$, so total revenue from good k, $y_{i,k}$, is given by

$$y_{i,k} = P_i\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}-1} b_{i,k}^{\eta} l_{i,k}^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

In equilibrium, each unit of resource will earn the same revenue no matter which good it is dedicated to producing. So, total revenue for location i, Y_i is given by

$$Y_i = \psi_y P_i x_i^{\eta} \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

where

$$\psi_y \equiv \psi_{g,\lambda}^{1-\eta}\sigma^{\frac{\sigma}{1-\sigma}} \frac{(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}}{1-\sigma}$$

Adapting the expression from section A.1 to account for the equilibrium fraction of land devoted to goods production, equilibrium wages are given by

$$w_i = \psi_{g,\lambda}^{1-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \eta (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} \left(\frac{\lambda_i}{x_i}\right)^{1-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

$$p_{i,\lambda} = \psi_{g,\lambda}^{-\eta} \sigma^{\frac{\sigma}{1-\sigma}} \eta (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} \left(\frac{\lambda_i}{x_i}\right)^{1-\eta} P_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

Note that $w_i x_i = \eta(1-\sigma)Y_i$ and $p_{i,\lambda}\psi_{g,\lambda}\lambda_i = (1-\eta)(1-\sigma)Y_i$–each factor is paid exactly its CES share of revenue, as expected.

A.3 Proof of housing consumption and equilibrium utility

From appendix A.1, the following relationship between the price of goods and the price of housing:

$$\left(\frac{p_{i,h}}{P_i}\right)^{\frac{\varphi}{1-\varphi}} = \left(\frac{\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}{\varphi^{\frac{\varphi}{1-\varphi}}(1-\varphi)}\right)^{\varphi} \psi_{g,\lambda}^{-\eta\varphi} x_i^{\eta\varphi} \lambda_i^{-\eta\varphi} \mathbb{M}_i^{\frac{1}{\chi}\left[\frac{\sigma}{1-\sigma}-\frac{\varphi}{1-\varphi}\right]\varphi}$$

Total housing production is then

$$H_i = \varphi^{\frac{\varphi}{1-\varphi}} \left(\frac{p_{i,h}}{P_i}\right)^{\frac{\varphi}{1-\varphi}} (1-\psi_{g,\lambda})\lambda_i \mathbb{M}_i^{\frac{1}{\chi}\frac{\varphi}{1-\varphi}}$$

$$= \left(\frac{\varphi\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}{1-\varphi}\right)^{\varphi} (1-\psi_{g,\lambda})\psi_{g,\lambda}^{-\eta\varphi} x_i^{\eta\varphi} \lambda_i^{1-\eta\varphi} \mathbb{M}_i^{\frac{1}{\chi}\frac{\varphi}{1-\sigma}}$$

Per-capita housing consumption, $h_i = \frac{H_i}{x_i}$, is then

$$h_i = \left(\frac{\varphi\sigma^{\frac{\sigma}{1-\sigma}}(1-\eta)\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}}{1-\varphi}\right)^{\varphi} (1-\psi_{g,\lambda})\psi_{g,\lambda}^{-\eta\varphi} \left(\frac{\lambda_i}{x_i}\right)^{1-\eta\varphi} \mathbb{M}_i^{\frac{1}{\chi}\frac{\varphi}{1-\sigma}}$$

Appendix A.1 also provides the following expression for C_i:

$$\begin{aligned} C_i &= \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i} \mathbb{M}_i^{\frac{1}{\chi}} \\ &= \alpha\sigma^{\frac{\sigma}{1-\sigma}}\kappa^{\frac{\kappa}{1-\sigma}}(1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\psi_{g,\lambda}^{-\eta}\left[\eta\psi_{g,\lambda}+1-\eta\right]\left(\frac{\lambda_i}{x_i}\right)^{1-\eta}\mathbb{M}_i^{\frac{1}{\chi(1-\sigma)}} \end{aligned}$$

Combining the expressions for per-capita consumption of housing and goods directly yields the following expression for equilibrium utility:

$$u_i = \psi_u \left(\frac{\lambda_i}{x_i}\right)^{\alpha(1-\eta)+(1-\alpha)(1-\eta\varphi)} \mathbb{M}_i^{\frac{\alpha+(1-\alpha)\varphi}{\chi(1-\sigma)}}.$$

Simplifying, this can be stated as

$$u_i = \psi_u \left(\frac{\lambda_i}{x_i}\right)^{1-\eta[\alpha+(1-\alpha)\varphi]} \mathbb{M}_i^{\frac{\alpha+(1-\alpha)\varphi}{\chi(1-\sigma)}},$$

where

$$\psi_u \equiv \alpha^\alpha \left(\sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}}\right)^{\alpha+\varphi(1-\alpha)} \left(\frac{\varphi(1-\eta)}{1-\varphi}\right)^{\varphi(1-\alpha)} (1-\psi_{g,\lambda})^{1-\alpha} \psi_{g,\lambda}^{-\eta[\alpha+\varphi(1-\alpha)]} [\eta\psi_{g,\lambda} + 1 -$$

A.4 Market Access

"Market access":

$$\mathbb{M}_i \equiv \left[\int_0^1 \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}} = \left[\int_0^A \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl + \int_A^1 \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}}$$

By definition, cost of production for a location-i producer of good k is $\frac{P_i}{s_{i,k}}$. Perfect competition implies that if good l is bought from location-j good l sold in location i will have a price equal to $p_{ij,l} = \frac{P_j}{s_{j,l}\gamma_{ji}}$.

The probability that the $p_{ij,l}$ is less than p, for $l \in [0, A]$, is given by

$$\begin{aligned}
\Pr(p_{ij,l} < p|\, l \in [0,A]) &= \Pr\left(\frac{P_j}{s_{j,l}\gamma_{ji}} < pl \in [0,A]\right) \\
&= \Pr\left(s_{j,l} > \frac{P_j}{p\gamma_{ji}} l \in [0,A]\right) = 1 - \Pr\left(s_{j,l} \leq \frac{P_j}{p\gamma_{ji}} l \in [0,A]\right) \\
&= 1 - e^{-\alpha_j \left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^\chi}
\end{aligned}$$

By the same reasoning,

$$\Pr(p_{ij,l} < p|\, l \in (A,1]) = 1 - e^{-m_j \left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^\chi}$$

Then, the probability that $p_{ij,l}$ is less than p, unconditional on whether l is an agricultural or non-agricultural good, can be calculated as

$$\begin{aligned}
\Pr\left(p_{ij,l} < p\right) &= 1 - \left[\prod_{l\in[0,A]} \left(1 - \Pr\left(p_{ij,l} < p | l \in [0,A]\right)\right)\right]\left[\prod_{l\in(A,1]} \left(1 - \Pr\left(p_{ij,l} < p | l \in (A,1]\right)\right)\right] \\
&= 1 - \left[\prod_{l\in[0,A]} e^{-\alpha_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right]\left[\prod_{l\in(A,1]} e^{-m_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right] \\
&= 1 - \left[e^{-\alpha_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right]^{A}\left[e^{-m_j\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}}\right]^{1-A} \\
&= 1 - e^{-\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}[A\alpha_j + (1-A)m_j]}
\end{aligned}$$

Then, by the properties of the Fréchet distribution, the probability that $p_{i,l} = \min_{j\in N} p_{ij,l}$ is less than p is given by

$$\begin{aligned}
\hat{G}_i(p) \equiv \Pr(p_{i,l} < p) &= 1 - \prod_{j\in N}\left[1 - \Pr(p_{ij,l} < p)\right] \\
&= 1 - \prod_{j\in N} e^{-\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}[A\alpha_j + (1-A)m_j]} \\
&= 1 - e^{-\sum_{j\in N}\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} p^{\chi}[A\alpha_j + (1-A)m_j]}
\end{aligned}$$

$$\frac{d\hat{G}_i(p)}{dp} = \chi p^{\chi-1}\mathbb{P}_i e^{-\mathbb{P}_i p^{\chi}}$$

with

$$\mathbb{P}_i \equiv \sum_{j\in N}\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi}[A\alpha_j + (1-A)m_j]$$

Market access is therefore given by

$$\begin{aligned}
\mathbb{M}_i &= \left[P_i^{\frac{\rho}{1-\rho}}\int_0^1\left(\frac{1}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} dl\right]^{\chi\frac{1-\rho}{\rho}} \\
&= \left[P_i^{\frac{\rho}{1-\rho}}\int_0^\infty p^{-\frac{\rho}{1-\rho}}\frac{d\hat{G}_i(p)}{dp} dp\right]^{\chi\frac{1-\rho}{\rho}} \\
&= \left[P_i^{\frac{\rho}{1-\rho}}\int_0^\infty p^{-\frac{\rho}{1-\rho}}\chi p^{\chi-1}\mathbb{P}_i e^{-\mathbb{P}_i p^{\chi}} dp\right]^{\chi\frac{1-\rho}{\rho}}
\end{aligned}$$

Change of variable: $x \equiv \mathbb{P}_i p^\chi$:

$$\mathbb{M}_i = \left[P_i^{\frac{\rho}{1-\rho}} \mathbb{P}_{i,a}^{\frac{1}{\chi}\frac{\rho}{1-\rho}} \int_0^\infty x^{1-\frac{1}{\chi}\frac{\rho}{1-\rho}-1} e^{-x} dx \right]^{\chi \frac{1-\rho}{\rho}}$$

Applying the definition of the gamma function, $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$:

$$\begin{aligned}
\mathbb{M}_i &= \Gamma\left(1 - \frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}} P_i^\chi \mathbb{P}_i \\
&= \Gamma\left(1 - \frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}} \sum_{j \in N} \left(\frac{P_i}{P_j}\right)^\chi \gamma_{ji}^\chi \left[A\alpha_j + (1-A)m_j\right]
\end{aligned}$$

Note that the previous steps require that the restriction $\chi > \frac{\rho}{1-\rho}$ holds.

A.5 Goods Market Clearing and Prices Derivation

The probability that location j exports a given good l to location i, r_{ji}, is the same as the probability that location j can provide good l at the lowest cost:

$$\begin{aligned}
r_{ji} &= \Pr\left(p_{i,j,l} < \min_{k \neq j}\{p_{i,k,l}\}\right) \\
&= \int_0^\infty \prod_{k \neq j} \left[1 - \Pr(p_{i,k,l} < p)\right] \frac{d\Pr\left(p_{i,j,l} < p\right)}{dp} dp \\
&= \int_0^\infty e^{-\sum_{k \in N} \left(\frac{P_k}{\gamma_{ki}}\right)^{-\chi} p^\chi [A\alpha_k + (1-A)m_k]} \chi p^{\chi-1} \left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} \left[A\alpha_j + (1-A)m_j\right] dp
\end{aligned}$$

Change of variable $x = \mathbb{P}_i p^\chi$:

$$\begin{aligned}
r_{ji} &= \frac{\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} \left[A\alpha_j + (1-A)m_j\right]}{\mathbb{P}_i} \int_0^\infty e^{-x} dx \\
&= \frac{\left(\frac{P_j}{\gamma_{ji}}\right)^{-\chi} \left[A\alpha_j + (1-A)m_j\right]}{\mathbb{P}_i}
\end{aligned}$$

In terms of market access:

$$r_{ji} = \Gamma\left(1 - \frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi\frac{1-\rho}{\rho}} \frac{\left(\frac{P_i}{P_j}\right)^\chi \gamma_{ji}^\chi \left[A\alpha_j + (1-A)m_j\right]}{\mathbb{M}_i}$$

Aggregate expenditure on good l in consumption:

$$
\begin{aligned}
x_i p_{i,l} c_{i,l} &= x_i \alpha \frac{w_i + p_{i,\lambda}\frac{\lambda_i}{x_i}}{P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} p_{il}^{-\frac{\rho}{1-\rho}} \\
&= \alpha \psi_{g,\lambda}^{-\eta} \left[\eta \psi_{g,\lambda} + 1 - \eta\right] \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} x_i^{\eta} \lambda_i^{1-\eta} P_i^{\frac{1}{1-\sigma}} \mathbb{M}_i^{\frac{(1-\rho)(\sigma-\rho)}{\chi(1-\sigma)}} p_{il}^{-\frac{\rho}{1-\rho}} \\
&= \alpha(1-\sigma) \left[\eta + \frac{1-\eta}{\psi_{g,\lambda}}\right] \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} \\
&= (1-\sigma) \frac{\alpha}{\alpha \varphi(1-\alpha)} \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}
\end{aligned}
$$

Aggregate expenditure on intermediate input l in goods production:

$$
\begin{aligned}
p_{i,l} z_{i,l} &= \frac{\sigma}{\eta(1-\sigma)} \frac{x_i w_i}{P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} p_{i,l}^{-\frac{1}{1-\rho}} \\
&= \psi_{g,\lambda}^{1-\eta} \frac{\sigma}{1-\sigma} \sigma^{\frac{\sigma}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} x_i^{\eta} \lambda_i^{1-\eta} P_i^{\frac{1}{1-\rho}} \mathbb{M}_i^{\frac{(1-\rho)(\sigma-\rho)}{\chi(1-\sigma)}} p_{i,l}^{-\frac{1}{1-\rho}} \\
&= \sigma \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}
\end{aligned}
$$

Aggregate expenditure on intermediate input l in housing production:

$$
\begin{aligned}
p_{i,l} z_{il,h} &= \frac{\varphi}{1-\varphi} \frac{p_{i,\lambda}}{p_{il}^{\frac{1}{1-\rho}} P_i^{-\frac{\rho}{1-\rho}} \mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} l_{i,h} \\
&= (1-\psi_{g,\lambda}) \psi_{g,\lambda}^{-\eta} (1-\eta) \frac{\varphi}{1-\varphi} \sigma^{\frac{\sigma}{1-\sigma}} \kappa^{\frac{\kappa}{1-\sigma}} (1-\sigma-\kappa)^{\frac{1-\sigma-\kappa}{1-\sigma}} x_i^{\eta} \lambda_i^{1-\eta} P_i^{\frac{1}{1-\sigma}} \mathbb{M}_i^{\frac{(1-\rho)(\sigma-\rho)}{\chi(1-\sigma)}} p_{i,l}^{-\frac{1}{1-\rho}} \\
&= \frac{1-\psi_{g,\lambda}}{\psi_{g,\lambda}} (1-\eta)(1-\sigma) \frac{\varphi}{1-\varphi} \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}} \\
&= (1-\sigma) \frac{\varphi(1-\alpha)}{\alpha + \varphi(1-\alpha)} \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}
\end{aligned}
$$

Aggregate expenditure on good l in location i for all purposes, as a function of its price:

$$
\tilde{y}_{i,l} \equiv x_i p_{i,l} c_{i,l} + p_{i,l} z_{i,l} + p_{i,l} z_{il,h} = \left(\frac{P_i}{p_{i,l}}\right)^{\frac{\rho}{1-\rho}} \frac{Y_i}{\mathbb{M}_i^{\frac{1}{\chi}\frac{\rho}{1-\rho}}}
$$

From this expression, it follows immediately that total aggregate expenditure on goods equals total aggregate revenue of goods-producing firms:

$$\tilde{Y}_i \equiv \int_0^1 \tilde{y}_{i,l} dl = Y_i$$

Now, using export probabilities r_{ji}, it is possible to calculate $\tilde{r}_{ji}$, the share of i's aggregate goods expenditure that is spent on goods from location j. As it turns out, $\tilde{r}_{ji} = r_{ji}$:

$$\tilde{r}_{ji} = \frac{\int_0^1 r_{ji} \tilde{y}_{i,l} dl}{\tilde{Y}_i} = r_{ji} \frac{Y_i}{\tilde{Y}_i} = r_{ji}$$

In equilibrium, aggregate revenue of goods producing firms in location $i \in N$ must equal total expenditure from all locations $j \in N$ on goods produced in i:

$$Y_i = \sum_{j \in N} r_{ij} Y_j$$

Now let us substitute in for r_{ij} and develop this expression a bit further:

$$P_i x_i^{\eta} \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}} = \Gamma \left(1 - \frac{1}{\chi}\frac{\rho}{1-\rho}\right)^{\chi \frac{1-\rho}{\rho}} \sum_{j \in N} \frac{\left(\frac{P_j}{P_i}\right)^{\chi} \gamma_{ij}^{\chi} \left[A\alpha_i + (1-A)m_i\right]}{\mathbb{M}_j} P_j x_j^{\eta} \lambda_j^{1-\eta} \mathbb{M}_j^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}}$$

$$\begin{aligned} P_i x_i^{\eta} \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1} & \sum_{j \in N} \left(\frac{P_i}{P_j}\right)^{\chi} \gamma_{ji}^{\chi} \left[A\alpha_j + (1-A)m_j\right] \\ &= \sum_{j \in N} \left(\frac{P_j}{P_i}\right)^{\chi} \gamma_{ij}^{\chi} \left[A\alpha_i + (1-A)m_i\right] P_j x_j^{\eta} \lambda_j^{1-\eta} \mathbb{M}_j^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1} \end{aligned}$$

With transitive asymmetry, i.e., if $\frac{\gamma_{ij}}{\gamma_{ji}}\frac{\gamma_{jk}}{\gamma_{kj}} = \frac{\gamma_{ik}}{\gamma_{ki}}$, and taking market access as given, the following is a solution to the system of equations implied by the preceding expression:

$$\frac{P_i x_i^{\eta} \lambda_i^{1-\eta} \mathbb{M}_i^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1}}{A\alpha_i + (1-A)m_i} \left(\frac{P_i}{P_j}\right)^{\chi} \gamma_{ji}^{\chi} = \left(\frac{P_j}{P_i}\right)^{\chi} \gamma_{ij}^{\chi} \frac{P_j x_j^{\eta} \lambda_j^{1-\eta} \mathbb{M}_j^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1}}{A\alpha_j + (1-A)m_j}$$

and, thus:

$$\left(\frac{P_i}{P_j}\right)^{1+2\chi} = \left(\frac{\gamma_{ij}}{\gamma_{ji}}\right)^{\chi} \frac{x_j^{\eta} \lambda_j^{1-\eta}}{x_i^{\eta} \lambda_j^{1-\eta}} \frac{A\alpha_i + (1-A)m_i}{A\alpha_j + (1-A)m_j} \left(\frac{\mathbb{M}_i}{\mathbb{M}_j}\right)^{\frac{1}{\chi}\frac{\sigma}{1-\sigma}-1}$$

If we apply the restriction that $\sigma = \frac{\chi}{1+\chi}$, then the above expression is a closed-form solution for relative prices for all locations. Given the types of values that are typically given to these parameters, in the literature, however, this is unlikely to be a reasonable restriction: it would imply, simultaneously, a very high share of intermediate inputs in production, and a very high elasticity of trade to transport costs. A smaller but still significant concern with this restriction is that it would also, due to the necessity that $\chi > \frac{\rho}{1-\rho}$, require a relatively high complementarity between goods. Assuming that $\sigma < \frac{\chi}{1+\chi}$, as is more reasonable, the interpretation of this expression is as follows: revenue per unit of input will be higher in locations that have less land and labor available for production, that have higher agricultural potential and technology levels, that have greater market access, and that face lower barriers to exporting than they do to importing.

A.5.1 One Period Spatial Equilibrium

In order to explore the basic properties of the mobility regime we have just specified, let us now define a one-period spatial equilibrium. Suppose the world exists for only a single period. A **one-period spatial equilibrium** consists of a static equilibrium summarized by u_i for all $i \in N$ and location choices by all consumers such that, given their starting locations, bilateral mobility costs, draws for idiosyncratic location preferences, and the location choices of other consumers, each consumer's choice maximizes his utility.

Following Redding (2016), the distribution of idiosyncratic preferences given by $M(.)$ implies that l_{ij}, the probability that a consumer with a starting location of i will choose to reside in j, will be given by the following:

$$l_{ij} = \frac{\vartheta_{ij}\mu_j^0 x_{j,b} u_j^{\varkappa}}{\sum\limits_{k\in N} \vartheta_{ik}\mu_k^0 x_{k,b} u_k^{\varkappa}}$$

$$x_j = \sum_{i\in N} l_{ij}x_{i,b} = \mu_j^0 x_{j,b} u_j^{\varkappa} \sum_{i\in N} \frac{\vartheta_{ij}x_{i,b}}{\sum\limits_{k\in N} \vartheta_{ik}\mu_k^0 x_{k,b} u_k^{\varkappa}} \tag{27}$$

These choice probabilities, aggregated over the distribution of starting populations $x_{i,b}$ for $i \in N$, imply the following ratios of basic utility that must hold for all $j, m \in N$:

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{\frac{x_j}{x_{j,b}}}{\frac{x_m}{x_{m,b}}} \frac{\sum\limits_{i\in N} \vartheta_{im}\tilde{l}_i}{\sum\limits_{i\in N} \vartheta_{ij}\tilde{l}_i} \right)^{\frac{1}{\varkappa}}, \tag{28}$$

where

$$\tilde{l}_i \equiv \frac{x_{i,b}}{\sum\limits_{k\in N} \vartheta_{ik}\mu_k^0 x_{k,b} u_k^{\varkappa}}.$$

The interpretation of this expression is the following: locations will have relatively higher utility in equilibrium which

1. have relatively low utility multipliers (i.e., $\mu_j^0 < \mu_m^0$),
2. are relatively costly for consumers in other locations to move to ($\sum_{i\in N} \vartheta_{ij}\tilde{l}_i < \sum_{i\in N} \vartheta_{im}\tilde{l}_i$), and
3. experience relatively larger inflows of resident population relative to their starting population ($\frac{x_j}{x_{j,b}} > \frac{x_m}{x_{m,b}}$).

The parameter $\varkappa$, then, determines the sensitivity of relative utilities to differences between locations of these three types. In the limit as $\varkappa$ approaches 1, a change in the ratio of amenity multipliers $\frac{\mu_m^0}{\mu_j^0}$ would be matched 1:1 by a change in relative utility $\frac{u_j}{u_m}$. In the opposite limit, as $\varkappa$ increases without bound, utility u_i is always equalized across locations in equilibrium regardless of the fundamentals.

Another interpretation of the parameter $\varkappa$ becomes apparent if we think about a *series* of one-period spatial equilibria indexed by t, such that $x_{i,b}(t+1) = x_i(t)$ for $t \in \{0, 1, 2, ...\}$. It can be shown that for an arbitrary distribution of starting population $x_{i,b}(0)$ or $i \in N$, such a series of equilibria is guaranteed to converge to a *stable* equilibrium, one in which $x_i(t) = x_{i,b}(t) = x_i$ for $i \in N$, as $t \to \infty$. $\varkappa$ determines the speed of this convergence, with a higher value implying faster convergence. In the limit as $\varkappa \to \infty$, the location choices of the very first equilibrium always yield the stable population distribution, regardless of the starting point.

To see how this specification of location preferences with idiosyncratic shocks nests the standard case of free mobility with no idiosyncratic shocks, consider a stable one-period spatial equilibrium. Consider the case in which all bilateral moving costs are zero: $\vartheta_{ij} = 1$ for all $i, j \in N$.[15] In this case, (28) implies that

$$\left(\mu_j^0\right)^{\frac{1}{\varkappa}} u_j = \left(\mu_m^0\right)^{\frac{1}{\varkappa}} u_m.$$

In other words, utility, controlling for location-specific amenity multipliers, is equalized. A spatial equilibrium under free mobility with no preference shocks would require exactly the same condition.

Now, to see how mobility restrictions between countries may play a role, let us consider the case where moving costs within each country are equal to zero, but moving costs between countries are infinite, as in the baseline model of Desmet, Nagy and Rossi-Hansberg (2016).[16] In this case, (28) implies that amenity-multiplier-controlled utility

[15]In the context of the paramaterization specified by (**??**), this requires that $\zeta_3 = 1$, $\zeta_4 = 0$, and $\bar{\vartheta}(l, m) = 1$ for all country pairs l, m.

[16]In the context of (**??**), this requires that $\zeta_3 = 1$, $\zeta_4 = 0$, and $\bar{\vartheta}(l, m)0$ for all country pairs l, m such that $l \neq m$.

must be equalized within countries, and also that preference shocks play no role in pinning down the inequalities in utility which may exist between countries.[17] If, alternatively, moving costs between countries are positive but finite, then preference shocks do play a role in determining relative utilities between countries, and $\varkappa$ again plays its role of deciding how large the equilibrium inequalities will be and how fast a series of equilibria will converge to the stable distribution.

A.6 Proof of parameter regions for forces of agglomeration and dispersion and long-run outcomes

Theorem 2 *Given the environment that has been described, if* $\frac{\nu_2}{\nu_1} = \eta$ *and the world enters a balanced growth path, utilities corresponding to* $\bar{u}$, $\tilde{\mathbf{U}}$ *and population levels corresponding to* $\mathbf{x}$ *must jointly satisfy the following three conditions:*

1. $\bar{u}^{\frac{1}{\nu_1}}$ *must be the largest eigenvalue and* $\mathbf{x}^{\{\eta\}}$ *must be the corresponding right eigenvector of the matrix* $\xi\psi_4^{\frac{1}{\nu_1}}\tilde{\mathbf{U}}^{-\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{\varsigma_m\}}\mathbf{\Lambda}^{1-\eta}$

2. *Given* $\mathbf{x}$ *and* $\bar{u}$, $\tilde{\mathbf{U}}$ *must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j) + \kappa} \frac{\sum_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$

   for all $j, m \in N$.

3. *The growth rate of population is equal to*

$$\varsigma_x = \frac{\sum_{i \in N} x_i \left[f_i^0 f(u_i) + \kappa \right]}{\sum_{i \in N} x_i} - 1,$$

   and so the growth rate of manufacturing potential is equal to

$$\varsigma_m = (1 + \varsigma_x)^{\eta} - 1$$

.

Corollary 2.1 *If* $\frac{\nu_2}{\nu_1} = \eta$, *and* $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, *and* $\mu_i^0 = \mu_j^0$ *for all* $i, j \in N$, *then, if the world enters a balanced growth path,* $u_i = u_j = \bar{u}$ *for all* $i, j \in N$ *and* $\bar{u}$ *and population levels corresponding to* $\mathbf{x}$ *are pinned down by the two following conditions:*

[17] The second point can be seen by noting that in a stable equilibrium, if ϑ_{ij} equals 0 whenever i and j belong to separate countries and 1 whenever they belong to the same country, and if j and m in (28) belong to separate countries, then (28) reduces to $1 = 1$, a condition which always holds and so cannot play a role in determining relative utilities between countries.

1. $\bar{u}^{\frac{1}{\nu_1}}$ *must be the largest eigenvalue and* $\mathbf{x}^{\{\eta\}}$ *must be the corresponding right eigenvector of the matrix* $\xi\psi_4^{\frac{1}{\nu_1}}\mathbf{\Lambda}^{\eta}\mathbf{\Theta}^{\{\varsigma_m\}}\mathbf{\Lambda}^{1-\eta}$.

2. *the growth rate of population must be equal to*

$$\varsigma_x = f^0 f(\bar{u}) + \kappa - 1,$$

and the growth rate of manufacturing potential equal to

$$\varsigma_m = (f^0 f(\bar{u}) + \kappa)^{\eta} - 1$$

.

The interpretation of this characterization is as follows: $\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{\mathbf{1}-\eta}$ is a matrix such that each ij^{th} element represents the access that the land area in location i which is being used to produce housing has to the land area in location j which is being used to produce goods. The largest eigenvalue of this matrix is, simply, a measure of how much land there is in the world and how well it is connected to other land. The dependence of the maximum utility level on this measure can be interpreted in the following way: land is a productive resource which is distributed across space, and people are better off when the locations holding this resource are better-connected. Similarly, the growth rate of the economy depends on this same measure: the economy grows faster when the world is better-connected.

The right eigenvector corresponding to the largest eigenvalue has in other contexts been interpreted as an *eigenvector centrality*, and this interpretation is appropriate here as well. This means that population agglomerates in locations that are *central*, in the sense of being well-connected, relative to the distribution of land.

Pre-multiplying the matrix $\mathbf{\Lambda}^{\eta}\mathbf{\Theta}\mathbf{\Lambda}^{\mathbf{1}-\eta}$ by $\tilde{\mathbf{U}}^{\frac{-1}{\nu_1}}$ applies weights according to locations' relative utilities, with higher weights being placed on locations with relatively low utility. This makes sense as, relative to the homogenous, free-mobility case in which utility u_i equalizes across locations, a location that has lower utility will have higher population and thus more productive capacity, again relative to the equalized-utility case.

In order to specify the condition which determines whether the world will achieve sustained growth in the long run or will instead converge to a steady, state, it is convenient to introduce the concept of a "hypothetical" population growth rate–the population growth rate which would obtain in a hypothetical balanced growth path with a specified growth rate of manufacturing potential.

Definition 3 *Let the* ***hypothetical balanced growth path population growth rate****,* $\tilde{\varsigma}_x(k)$*, be defined implicitly as a function of* k *by the following three conditions:*

1.

$$\tilde{\varsigma}_x(k) = \frac{\sum_{i \in N} x_i \left[f_i^0 f(u_i) + \kappa\right]}{\sum_{i \in N} x_i} - 1$$

2. Given $\mathbf{x}$ *and* $\bar{u}$, $\tilde{\mathbf{U}}$ *satisfies the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j) + \kappa} \frac{\sum_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$

for all $j, m \in N$.

3. $\bar{u}^{\frac{1}{\nu_1}}$ *is the largest eigenvalue and* $\mathbf{x}^{\{\eta\}}$ *is the corresponding right eigenvector of the matrix* $\xi \psi_4^{\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \mathbf{\Theta}^{\{k\}} \mathbf{\Lambda}^{1-\eta}$

Mirroring corollary 7.1, in the case where $\frac{\nu_2}{\nu_1} = \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, the definition of $\tilde{\varsigma}_x(k)$ given by definition 3 simplifies considerably, and depends on only two distinct conditions:

1.

$$\tilde{\varsigma}_x(k) = f^0 f(\bar{u}) + \kappa - 1$$

2. $\bar{u}^{\frac{1}{\nu_1}}$ is the largest eigenvalue of the matrix $\xi \psi_4^{\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \mathbf{\Theta}^{\{k\}} \mathbf{\Lambda}^{1-\eta}$.

In any case, the condition for long-run sustained growth is given by the following theorem:

Theorem 3 *Given the environment that has been described, if* $\frac{\nu_2}{\nu_1} = \eta$, *the world will asymptotically approach a unique balanced growth path if and only if* $\tilde{\varsigma}_x(0) > 0$.

Proof: See Appendix **??**.

Theorem 3 makes clear the dependence of growth on the level of connectedness: if transport costs are high enough, and thus the largest eigenvalue of $\mathbf{\Lambda}^{\eta} \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta}$ is small enough, sustained growth is not possible, and the economy stagnates instead. Low-enough transport costs are a necessary condition for sustained growth.

Now let us examine the allocations of this steady state economy.

Theorem 4 *In the environment that has been described, if* $\frac{\nu_2}{\nu_1} = \eta$ *and the world converges to a Malthusian steady state, utilities corresponding to* $\bar{u}$, $\tilde{\mathbf{U}}$ *and population levels corresponding to* $\mathbf{x}$ *must jointly satisfy the following three conditions:*

1. $\dfrac{\sum_{i \in N} x_i \left[f_i^0 f(\bar{u}\tilde{u}_i) + \kappa\right]}{\sum_{i \in N} x_i} = 1$

2. $\mathbf{x}^{\{\eta\}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \left(\mathbf{I} - \xi \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \mathbf{\Theta} \mathbf{\Lambda}^{1-\eta} \right)^{-1} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \boldsymbol{\alpha}$

3. Given $\mathbf{x}$ *and* $\bar{u}$*,* $\tilde{\mathbf{U}}$ *must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa \sum_{i \in N} \vartheta_{im} \tilde{l}_i}{f_j^0 f(u_j) + \kappa \sum_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$

for all $j, m \in N$*.*

Corollary 4.1 *If* $\frac{\nu_2}{\nu_1} = \eta$*, and* $\vartheta_{ij} = 1$*,* $f_i^0 = f_j^0 = f^0$*, and* $\mu_i^0 = \mu_j^0$ *for all* $i, j \in N$*, and the world converges to a Malthusian steady state, then*

- $u_i = u_j = \bar{u} = f^{-1}\left(\frac{1-\kappa}{f^0}\right)$
- $\mathbf{x}^{\{\eta\}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \left(\mathbf{I} - \xi \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \mathbf{\Theta} \mathbf{\Lambda}^{1-\eta} \right)^{-1} \mathbf{\Lambda}^{\eta} \boldsymbol{\alpha}$

Theorem 8 shows that the centrality interpretation of the equilibrium population distribution can be maintained in the steady state as well as in the balanced growth path, except that in this case it is not eigenvector centrality but the closely-related *Katz-Bonacich centrality.*[18] In a steady state, the maximum utility $\bar{u}$ is exactly at the level that is necessary for there to be zero population growth–in the simpler case treated by corollary 8.1 where utility equalizes across locations, this may be thought of as the "subsistence" level of utility.[19]

The characterization of balanced growth path and steady state allocations, as well as the condition that determines which type of allocation is the long-run destination of the economy, are similar for the case where $\frac{\nu_2}{\nu_1} < \eta$ and the forces of agglomeration are stronger than those of dispersion. The most important difference is that unlike in the previous case, where long-run utility for any single location was strictly decreasing in its own population, now the relationship is non-monotonic, with a downward-sloping portion followed by a an upward-sloping, concave portion. This means that even if transport costs are very high, so that an economy starting from nothing would quickly stagnate, a

[18]See, for example, Bonacich (1987).

[19]If we label the potential balanced growth path utility level as determined by the matrix $\mathbf{\Lambda}^{\eta} \mathbf{\Theta} \mathbf{\Lambda}^{1-\eta}$ as $\bar{u}_b$ and the "subsistence" level of utility as $\bar{u}_s$, the relation of the steady state to the balanced growth path can be illustrated in the following way. If $\bar{u}_s > \bar{u}_b$, i.e., if potential balanced growth path utility is lower than the level necessary to sustain growth, then by corollary **??** the world converges to a steady state, and also the matrix $\mathbf{I} - \xi \psi_4^{\frac{1}{\nu_1}} \bar{u}_s^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \mathbf{\Theta} \mathbf{\Lambda}^{1-\eta}$ is guaranteed to be invertible. As $\bar{u}_b \to \bar{u}_s$ from below, i.e., as transport costs become lower, the matrix $\mathbf{I} - \xi \psi_4^{\frac{1}{\nu_1}} \bar{u}_s^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\eta} \mathbf{\Theta} \mathbf{\Lambda}^{1-\eta}$ will come closer and closer to being singular. If $\bar{u}_b \geq \bar{u}_s$, then the population distribution implied by corollary 8.1 either does not exist or has negative elements–meaning that the only stable long-run outcome is the balanced growth path, with $\bar{u} = \bar{u}_b$, and allocations as given by corollary 7.1.

sustained-growth outcome can always be achieved if only the starting levels of technology and population are above a certain threshold.

In order to express this condition succinctly, it is convenient to define the population growth rate along a hypothetical transition path as a function of the population level in every location. It is convenient to abstract from the gradual adjustment of population in this hypothetical transition path, and assume that the population distribution in each period corresponds to the *stable distribution* associated with that level of total world population, where the *stable distribution* is defined formally as follows:

Definition 4 *Let the* ***stable distribution*** *associated with a total population level* $\bar{x} = \sum_{i \in N} x_i$ *be defined as a distribution such that* $x_{i,b} = x_i$.

In this hypothetical transition path, it is also convenient to abstract from the gradual gradual accumulation of ideas, and assuming that levels of technology instantly jump to the long-run levels associated with the stable population distribution.

Definition 5 *Let the* ***hypothetical transition path population growth rate****,* $\hat{\varsigma}_x(\bar{x}, t)$*, be defined implicitly as a function satisfying the following conditions:*

1. $\sum_{i \in N} x_i = \bar{x}$ *and the population distribution is* ***stable****.*

2. $$\tilde{\varsigma}_x(\bar{x}) = \frac{\sum_{i \in N} x_i \left[f_i^0 f(u_i) + \kappa\right]}{\sum_{i \in N} x_i} - 1$$

3. $$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}\right]$$

4. *Given* $\mathbf{x}$ *and* $\bar{u}$*,* $\tilde{\mathbf{U}}$ *must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j(t)) + \kappa} \frac{\sum_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum_{i \in N} \vartheta_{ij} \tilde{l}_i}\right)^{\frac{1}{\varkappa}}$$

for all $j, m \in N$*.*

In the special case where $\frac{\nu_2}{\nu_1} = \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, the conditions given in definition 5 are reduced to three:

1. $\sum_{i \in N} x_i = \bar{x}$ and the population distribution is *stable*

2. $$\tilde{\varsigma}_x(\bar{x}) = f^0 f(\bar{u}) + \kappa - 1$$

3.
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta} \right]$$

Now let us define a *critical population level* as the threshold such that if the world starts with a stable population distribution and long-run levels of technology, and the total population level is higher than this critical level, it will achieve sustained growth.

Definition 6 *Considering the set of long-run, stable population and technology distributions, let the* ***critical population level*** $\bar{x}^*$ *be defined as follows for the following two cases:*

Case 1: *If* $\min_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} > 0$*, then* $\bar{x}^* = 0$

Case 2: *If* $\min_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} \leq 0$*, then* $\bar{x}^*$ *is the point such that* $\hat{\varsigma}_x(\bar{x}) = 0$ *and* $\frac{\partial \hat{\varsigma}_x(\bar{x})}{\partial \bar{x}} > 0$.

Making use of definition 6, the following theorem provides sufficient conditions for the economy to stagnate into a steady state in the long run.

Theorem 5 *Given the environment that has been described, if* $\frac{\nu_2}{\nu_1} < \eta$*, the following two conditions are sufficient for the world to converge to a Malthusian steady state in the long run:*

1. *The initial distribution of population* $\mathbf{x}(0)$ *is stable, with total population given by* $\bar{x}(0)$*, and the initial levels of manufacturing potential in each location are the long-run levels associated with* $\mathbf{x}(0)$*.*

2. $\bar{x}(0) \leq \bar{x}^*$

The implications of theorem 5 have an intuitive interpretation: if transportation costs are low enough, the economy will achieve sustained growth in the long run regardless of its starting point. If, however, transportation costs are high enough such that $\min_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} \leq 0$, then unless initial levels of population and technology are above a certain threshold, which for the case of stable, long-run starting conditions is given by $\bar{x}^*$, the economy will stagnate.

A full characterization of the necessary conditions for stagnation requires a consideration of all possible starting points for the economy, including population distributions that are not stable and arbitrary distributions of manufacturing potential.

Theorem 6 *Define* $\mathbf{s}(0)$ *as an* $n^3(n-1)$*-dimensional vector in* $\mathbb{R}_+^{n^3(n-1)}$ *composed of* $x_i(0)$*,* $m_{i,I}(0)$*,* $m_{ij,D}(0)$ *for* $i \in N$ *and* $j \neq i$*. There exists a function* $z(\mathbf{s})$ *in* $n^3(n-1)$ *arguments and a* $n^3(n-1)$*-dimensional hypersurface defined by the condition* $z(\mathbf{s}) = 0$ *such that the economy will converge to a Malthusian steady state if and only if* $z(\mathbf{s}(0)) \geq 0$*.*

Proof: See Appendix **??**.

What theorem 6 says is that there exists a frontier of initial population and technology levels such that, if initial levels lay within the limits of that frontier, the economy stagnates in the long run. In the case where transport costs are low enough that $\min_{\bar{x}} \{\hat{\varsigma}_x(\bar{x})\} > 0$, this frontier collapses to the origin: $z(\mathbf{s}) = 0$ for all $\mathbf{s} \in \mathbb{R}_+^{n^3(n-1)}$. This theorem is also valid when $\frac{\nu_2}{\nu_1} \geq \eta$, though it is obviously not as useful for analyzing these cases as the preceding theorems. In cases where $\frac{\nu_2}{\nu_1} > \eta$, for example, the frontier defined by $z(\mathbf{s})$ expands outward from the origin without bound such that $z(\mathbf{s}) \geq 0$ for all $\mathbf{s} \in \mathbb{R}_+^{n^3(n-1)}$.

Theorem 7 *Given the environment that has been described, if $\frac{\nu_2}{\nu_1} < \eta$ and the world enters a balanced growth path, utilities corresponding to $\bar{u}$, $\tilde{\mathbf{U}}$ and population levels corresponding to $\mathbf{x}$ must jointly satisfy the following three conditions:*

1.
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \xi \mathbf{\Theta}^{\{\varsigma_m\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}$$

2. *Given $\mathbf{x}$ and $\bar{u}$, $\tilde{\mathbf{U}}$ must satisfy the system of equations given by*
$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 \bar{f} + \kappa}{f_j^0 \bar{f} + \kappa} \frac{\sum_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$
for all $j, m \in N$.

3. *The growth rate of population is equal to*
$$\varsigma_x = \frac{\sum_{i \in N} x_i \left[f_i^0 \bar{f} + \kappa \right]}{\sum_{i \in N} x_i} - 1,$$
and so the growth rate of manufacturing potential is equal to
$$\varsigma_m = (1 + \varsigma_x)^{\eta} - 1$$

.

Corollary 7.1 *If $\frac{\nu_2}{\nu_1} = \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, then, if the world enters a balanced growth path, $u_i = u_j = \bar{u}$ for all $i, j \in N$ and $\bar{u}$ and population levels corresponding to $\mathbf{x}$ are pinned down by the following single condition:*

1.
$$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \xi \mathbf{\Theta}^{\{\varsigma_m\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}$$

where the growth rate of population is equal to
$$\varsigma_x = f^0 \bar{f} + \kappa - 1,$$

and the growth rate of manufacturing potential equal to

$$\varsigma_m = (f^0 \bar{f} + \kappa)^{\eta} - 1$$

.

Theorem 8 *In the environment that has been described, if either $\frac{\nu_2}{\nu_1} > \eta$ or $\frac{\nu_2}{\nu_1} < \eta$ and the world converges to a Malthusian steady state, utilities corresponding to $\bar{u}$, $\tilde{\mathbf{U}}$ and population levels corresponding to $\mathbf{x}$ must jointly satisfy the following three conditions:*

1. $\dfrac{\sum\limits_{i \in N} x_i \left[f_i^0 f(\bar{u}\tilde{u}_i) + \kappa\right]}{\sum\limits_{i \in N} x_i} = 1$

2. $$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \tilde{\mathbf{U}}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}\right]$$

3. *Given $\mathbf{x}$ and $\bar{u}$, $\tilde{\mathbf{U}}$ must satisfy the system of equations given by*

$$\frac{u_j}{u_m} = \left(\frac{\mu_m^0}{\mu_j^0} \frac{f_m^0 f(u_m) + \kappa}{f_j^0 f(u_j) + \kappa} \frac{\sum\limits_{i \in N} \vartheta_{im} \tilde{l}_i}{\sum\limits_{i \in N} \vartheta_{ij} \tilde{l}_i} \right)^{\frac{1}{\varkappa}}$$

   for all $j, m \in N$.

Corollary 8.1 *If either $\frac{\nu_2}{\nu_1} > \eta$ or $\frac{\nu_2}{\nu_1} < \eta$, and $\vartheta_{ij} = 1$, $f_i^0 = f_j^0 = f^0$, and $\mu_i^0 = \mu_j^0$ for all $i, j \in N$, and the world converges to a Malthusian steady state, then*

- $u_i = u_j = \bar{u} = f^{-1}\left(\frac{1-\kappa}{f^0}\right)$

- $$\mathbf{x}^{\frac{\nu_2}{\nu_1}} = \psi_4^{\frac{1}{\nu_1}} \bar{u}^{-\frac{1}{\nu_1}} \mathbf{\Lambda}^{\frac{\nu_2}{\nu_1}} \left[\boldsymbol{\alpha} + \xi \mathbf{\Theta}^{\{0\}} \mathbf{\Lambda}^{1-\eta} \mathbf{x}^{\eta}\right]$$